Numerical simulation of functionally graded plane elastic medium by finite superelement method

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Abstract

A numerical model of a continuously inhomogeneous elastic medium is constructed by using the finite superelement method. The superelement is developed by meshless method and based on Bernstein polynomials. A series of elasticity problems is considered for different continuously inhomogeneous media under mixed boundary conditions; stress-strain states are obtained.

Keywords: finite superelement, Bernstein polynomials, elasticity, continuously inhomogeneous medium.

1. Introduction

A numerical model is proposed to describe the behavior of a continuously inhomogeneous medium. The model is based on the method of finite superelements [2]. A computer code is developed that implements the said numerical model and algorithm.

The model allows calculating of mechanical properties of functionally graded, porous, and other inhomogeneous media, which are now actively studied and used in new technologies.

The proposed model uniformly describes arbitrary physical structure of the superelement, without meshing for each individual superelement (meshless method [1] is used to calculate the structure of the superelement).

Using the developed method, the elasticity problem for a square region under the influence of the vertical pressure is solved for several media.

2. Numerical method

The method is based on the finite superelement method with the following modifications. All functions, including components of the tensor of elasticity, are presented in the form of linear combinations of Bernstein polynomials [3] of given order. This allowed separating the displacements functions on the carcass functions (known functions satisfying the boundary conditions) and the kernel functions (unknown functions equal to zero on the boundary).

3. Statement of problem

A square elastic region is considered. The following boundary conditions are given: the bottom left corner is fixed; at the bottom side the horizontal loading is zero, and vertical displacement is zero; the left and right sides are free from loadings; the top side is under uniform pressure p (Figure 1).

The following media are considered: a homogeneous medium, a linear functionally graded medium, a medium with a distributed inclusion, a medium with central damage area.

4. Results

Several problems of the theory of elasticity are solved using the proposed method. The appropriate superelements are built, fields of stresses and deformations are derived (Figures 2, 3).

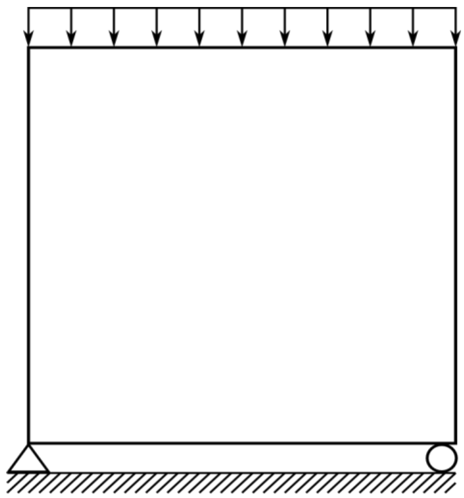

Figure 1: Statement of problem

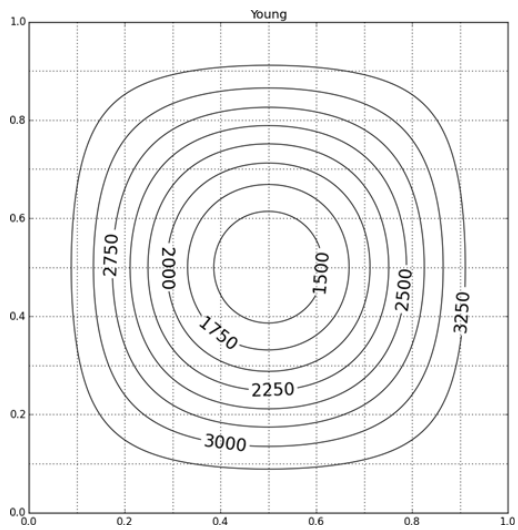

Figure 2: Young's modulus [MPa] of the superelement containing a central damage region

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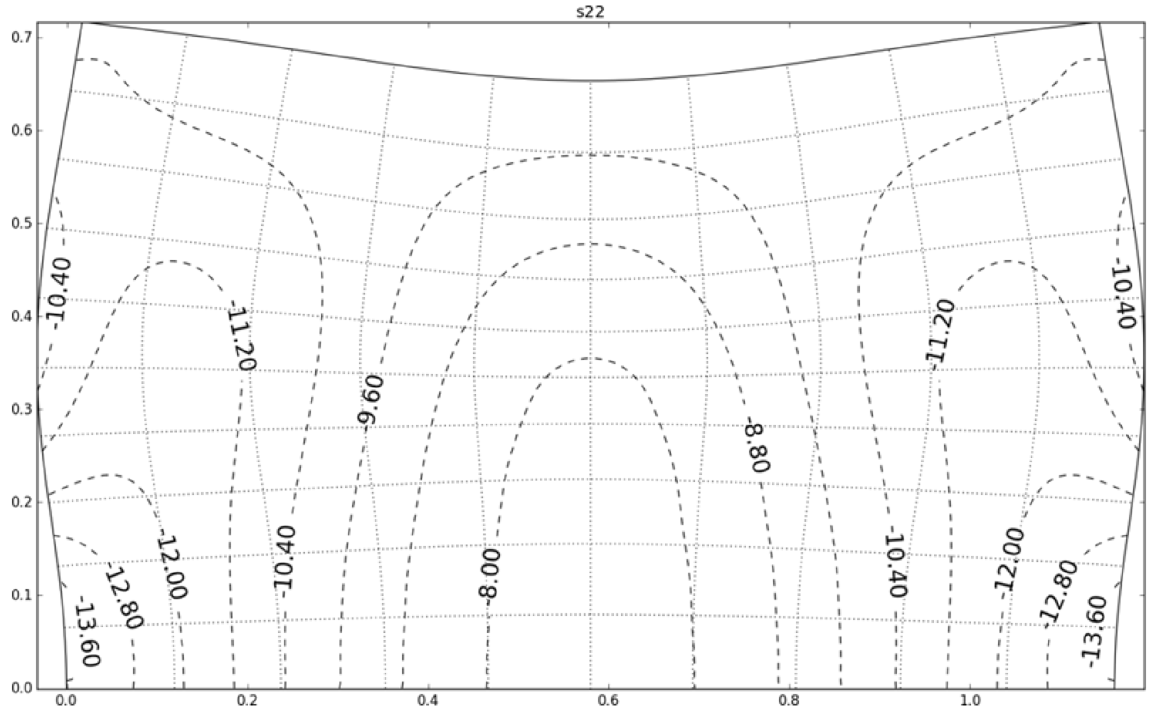

Figure 3: Stress σ_{22} [MPa] of the superelement containing a central damage region (vertical loading p = 10MPa).

5. Conclusion

The developed superelement allows solving of the plane elasticity problem for a square continuously inhomogeneous region. Elasticity moduli of the medium have a polynomial distribution. The boundary conditions of any kind should also be described by polynomial functions.

The developed code implements the proposed numerical algorithm. It allows calculating the generalized stiffness matrix of the superelement. It also allows solving the problem of elasticity corresponding to the superelement with the given boundary conditions.

The code can be applied for modeling of functionally graded, porous and other heterogeneous media. The proposed numerical algorithm allows modifying and expanding the basis, in order to improve accuracy and efficiency of calculation of stress-strain state of a given heterogeneous medium, taking into account its features.

References

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