Efficiency and accuracy investigation of the Craig-Bampton method through continuum vibration tests

Péter Horváth

Department of Applied Mechanics, Széchenyi István University
Egyetem Tér 1, 9026 Győr, Hungary
e-mail: horvpe@sze.hu

Abstract

The paper investigates and shows the efficiency and accuracy of the Craig-Bampton model order reduction method on the analysis of a cantilever beam and rod with harmonic excitation. The results of different finite element- and Craig-Bampton models are compared to the analytic continuum vibration results as reference.

Keywords: model order reduction, Craig-Bampton method, continuum vibration, frequency response function

1. Introduction

In the engineering praxis, the aim for better computation accuracy and shorter calculation time of dynamics problems are even higher nowadays. To achieve this aim, a model order reduction (MOR) method can be used. Using a MOR method, the dimension of equation-system of motion can be substantially decreased, while the significant characteristics of the system remain. In the literature, e.g. in Ref [4], a lot of MOR methods can be found, out of which, the most widespread in the industry is the Craig-Bampton method, which is a combination of the static- and modal reduction. This paper introduces this method, and shows its efficiency and accuracy on the vibration analysis of a cantilever beam and rod.

2. Problem description

To demonstrate the efficiency and accuracy of the Craig-Bampton method, let's consider a cantilever beam (case 1) and a rod (case 2), acting a vertical (case 1), and axial (case 2) concentrated force on it, shown in Fig. 1.

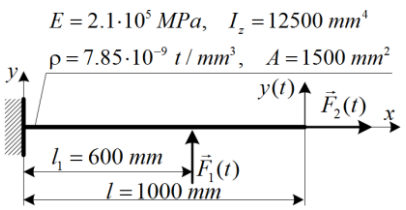

Figure 1: The beam and rod model

The objects of the analysis are as follows:

1) To apply the Craig-Bampton MOR method on the free-free beam (case 1) and rod (case 2) and compare its results (natural frequencies) with the results provided by the FEM and analytic methods.
2) Creation of a cantilever beam (case 1) and rod (case 2) by applying a fixed support on the left end of the beam and compare the results of the different methods.
3) To apply a harmonic excitation ($\vec{F}_1$ in case 1 and $\vec{F}_2$ in case 2) and observe the amplitudes of the vibration at the end of the beam or rod (e.g. plotting the transfer function)

3. Application of the Craig-Bampton method

For the application of the Craig-Bampton method, the degrees-of-freedom of the beam's FEM model have to be partitioned into master (*m*) and slave (*s*) degrees-of-freedom. Hence the partitioned equation-system of motion looks as follows:

$$\begin{bmatrix} \underline{\underline{M}}_{mm} & \underline{\underline{M}}_{ms} \\ \underline{\underline{M}}_{sm} & \underline{\underline{M}}_{ss} \end{bmatrix} \begin{bmatrix} \ddot{\underline{q}}_m \\ \ddot{\underline{q}}_s \end{bmatrix} + \begin{bmatrix} \underline{\underline{K}}_{mm} & \underline{\underline{K}}_{ms} \\ \underline{\underline{K}}_{sm} & \underline{\underline{K}}_{ss} \end{bmatrix} \begin{bmatrix} \underline{q}_m \\ \underline{q}_s \end{bmatrix} = \begin{bmatrix} \underline{f}_m \\ \underline{f}_s \end{bmatrix}, \tag{1}$$

The reduced system matrices can be achieved by a transformation using the Craig-Bampton transformation matrix, introduced in Ref. [1].

$$\underline{\underline{T}}_{CB} = \begin{bmatrix} \underline{\underline{I}}_{mm} & \underline{\underline{0}}_{mp} \\ \underline{\underline{R}}_{sm} & \underline{\underline{\Phi}}_{sp} \end{bmatrix} = \begin{bmatrix} \underline{\underline{I}}_{mm} & \underline{\underline{0}}_{mp} \\ -\underline{\underline{K}}_{ss}^{-1}\underline{\underline{K}}_{sm} & \underline{\underline{\Phi}}_{sp} \end{bmatrix}. \tag{2}$$

The left side column of the above matrix contains the static modes, while the right side contains the fixed boundary normal modes, where $\underline{\underline{\Phi}}_{sp}$ is the reduced modal matrix - containing *p* modes - of the slave system. The modes of this modal matrix are gained from the following eigenvalue problem:

$$\left(\underline{\underline{K}}_{ss} - \alpha_{ss}^2 \underline{\underline{M}}_{ss}\right)\underline{\underline{Q}}_{ss} = \underline{\underline{0}}. \tag{3}$$

The natural frequencies and natural modes of the Craig-Bampton model are gained from the following eigenvalue problem:

$$\left(\underline{\underline{K}}_r - \alpha_i^2 \underline{\underline{M}}_r\right)\underline{\underline{Q}}_i = \underline{\underline{0}}, \tag{4}$$

where $\underline{\underline{K}}_r$ and $\underline{\underline{M}}_r$ are the reduced stiffness and mass matrices.

In case 1, the master degrees-of-freedom are the vertical and angular displacements of the node at the left end, the vertical displacement of the node at l_1 location and the vertical displacement of the node at the right end of the beam, as can be seen in details in Ref. [3]. In case 2 the master degrees-of-freedom are the axial displacements of the nodes at the left and at the right end of the rod.

4. Numerical results

The results of the first two tests i.e. the natural frequencies of the free-free end and fixed-free end beam (case 1) and rod (case 2) are summarized in Table 1. The investigations were carried out on the analytic model, on different FEM and CB models, the characteristics of which are also indicated in Table 1.

Table 1: Comparison of the natural frequencies

model name:		A	B	C	D	E	F	G
type of the model		analytic	FEM	CB	FEM	CB	CB	CB
original FEM modell dimension		-	22	22	12	22	42	22
physical+modal coordinates		-	22+0	4+18	12+0	4+8	4+8	4+4
	1	159.5	159.5	159.5	159.6	159.5	159.5	159.5
	2	439.7	439.8	439.8	441.1	439.8	439.7	441.7
	3	861.9	862.7	862.7	870.2	862.8	862.1	865.2
	4	1424.8	1428.2	1428.2	1439.0	1429.2	1426.7	1451.7
CASE 1:	5	2128.4	2139.2	2139.2	2368.0	2141.3	2131.8	2263.0
Free - Free Beam	6	2972.7	3000.2	3000.2	3409.1	3018.4	3001.2	5729.1
	7	3957.7	4016.5	4016.5	4862.7	4061.6	4026.7	-
	8	5083.5	5187.7	5187.7	6875.9	5230.7	5147.6	-
	9	6350.0	6444.6	6444.6	10376.2	6543.0	6549.5	-
	10	7757.2	8558.4	8558.4	10902.3	18323.1	18290.6	-
type of the model		analytic	FEM	CB	FEM	CB	CB	CB
original FEM modell dimension		-	20	20	10	20	40	20
physical+modal coordinates		-	20+0	2+18	10+0	2+8	2+8	2+4
	1	25.1	25.1	25.1	25.1	25.1	25.1	25.1
	2	157.1	157.1	157.1	157.2	157.1	157.1	157.1
	3	439.8	439.9	439.9	441.4	440.0	439.9	440.9
	4	861.9	862.7	862.7	872.0	862.8	862.1	864.0
CASE 1:	5	1424.8	1428.4	1428.4	1447.3	1429.0	1426.0	1456.6
Fixed - Free Beam	6	2128.4	2139.9	2139.9	2404.4	2141.8	2131.2	2286.0
	7	2972.7	3002.3	3002.3	3516.5	3019.9	2999.2	-
	8	3957.7	4022.3	4022.3	5099.6	4065.9	4023.1	-
	9	5083.5	5200.8	5200.8	7244.4	5239.2	5135.0	-
	10	6350.0	6465.4	6465.4	10656.9	6566.9	6561.0	-
type of the model		analytic	FEM	CB	FEM	CB	CB	CB
original FEM modell dimension		-	21	21	11	21	41	21
physical+modal coordinates		-	21+0	2+19	11+0	2+9	2+9	2+4
	1	2586.1	2588.8	2588.8	2596.7	2589.0	2587.2	2591.0
	2	5172.2	5193.5	5193.5	5257.6	5194.9	5179.9	5236.0
	3	7758.3	7830.3	7830.3	8047.6	7838.3	7788.3	7911.9
	4	10344.4	10515.3	10515.3	11030.4	10528.2	10408.3	12278.3
CASE 2:	5	12930.5	13264.9	13264.9	14257.9	13313.2	13080.6	15493.9
Free - Free Rod	6	15516.6	16095.3	16095.3	17740.8	16154.4	15747.9	-
	7	18102.7	19022.4	19022.4	21380.4	19238.4	18593.2	-
	8	20688.8	22060.8	22060.8	24850.7	22297.5	21329.6	-
	9	23274.9	25222.7	25222.7	27499.8	28795.6	28085.4	-
	10	25861.0	28515.8	28515.8	28515.8	32220.5	31308.2	-
type of the model		analytic	FEM	CB	FEM	CB	CB	CB
original FEM modell dimension		-	20	20	10	20	40	20
physical+modal coordinates		-	20+0	1+19	10+0	1+9	1+9	1+4
	1	1293.0	1293.4	1293.4	1294.4	1293.4	1293.2	1293.6
	2	3879.1	3888.1	3888.1	3915.1	3888.5	3882.0	3894.2
	3	6465.2	6506.9	6506.9	6632.4	6508.7	6478.5	6540.3
	4	9051.3	9165.7	9165.7	9511.2	9171.1	9088.3	9296.6
CASE 2:	5	11637.4	11881.0	11881.0	12611.4	11894.0	11717.4	14042.7
Fixed - Free Rod	6	14223.5	14669.0	14669.0	15969.9	14696.8	14373.6	-
	7	16809.6	17545.8	17545.8	19554.4	17602.8	17067.7	-
	8	19395.7	20526.8	20526.8	23169.7	20644.3	19819.4	-
	9	21981.8	23625.7	23625.7	26330.1	23892.7	22681.0	-
	10	24567.9	26852.6	26852.6	28254.5	30680.0	29802.7	-

Observing the table, the following consequences can be drawn:

- Comparing B and D *FEM* models, one can observe, that using a finer mesh increases the accuracy of the calculated natural frequencies.
- As it can be seen from the B *FEM* and C *CB* models, if the dimension of the equation-system of motion is not decreased during the *CB* transformation, namely only the physical coordinates are transformed into a set of physical and modal coordinates, the *FEM* and *CB* models provide the same results.
- The D *FEM* and E *CB* models have the same dimension, thus, the numerical cost is the same. Regarding the accuracy, the results of the *CB* model are by far closer to the exact analytical results, than the results provided by the E *FEM* model. This tendency changes only at higher frequencies, where the *FEM* model gives better results.
- The E and F *CB* models have the same dimension, but the F model is derived from a finer *FEM* model. Comparing the results with the analytic results, one can find out that using a finer mesh for the *CB* model reduction increases the accuracy, but only with a minor extent.
- Comparing the G *CB* model with the D *FEM* model, one can observe, that at lower frequencies, the *CB* model provides better accuracy at lower frequencies despite the smaller dimension number.

For the third test - the excited analysis - the frequency response functions (*FRF*) of the A, D and F models were compared. According to Ref. [2], the *FRF* is described by the following formula

$$H(\omega) = \sum_{i=1}^{n} \frac{Q_i(l)Q_i(l)}{\left(\alpha_i^2 - \omega^2\right) + j2\zeta_i\alpha_i\omega}. \tag{5}$$

Figure 2 illustrates the comparison of the A analytical, D *FEM*, and F *CB* models, using $\zeta = 0.01$ damping ratio. As it can be observed from the figure, both in case of the beam and in case of the rod, the *FRF* of the F model provides a quite good agreement with the analytic solution, while the D model shifts into higher frequencies. The F model gives worse results only at high frequencies.

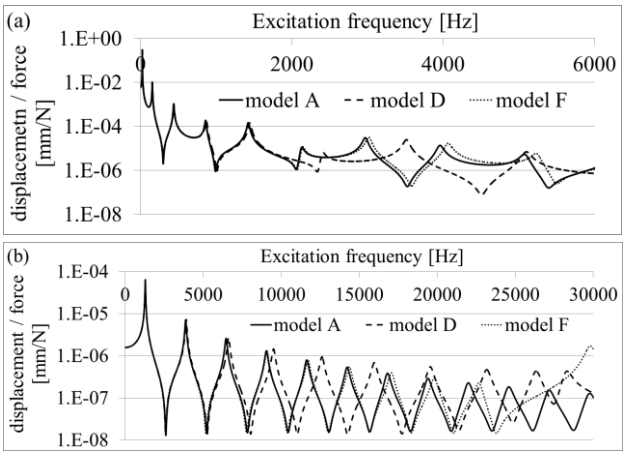

Figure 2: Comparison of the frequency response functions in case 1 (a) and case 2 (b)

References

[1] Craig, R. and Bampton, M., Coupling of Substructures for Dynamic Analyses, *AIAA Journal*, 6(7), pp 1313–1319, 1968.

[2] Heylen, W., Lammens, S. and Sas, P., *Modal Analysis Theory and Testing*, Katholieke Universiteit Leuven, Belgium, 1998.

[3] Horváth, P. and Égert, J., Continuum beam vibration efficiency test of the Craigh-Bampton method, *GÉP* 67:(5-6) pp. 108-112 (in Hungarian), 2016.

[4] Nowakowski, C., Kürcshner, P., Eberhard, P. and Benner, P., Model Reduction of an Elastic Crankshaft for Elastic Multibody Simulations, *Zeitschrift für angewandte Mathematik und Mechanik*, 93, pp 198-216, 2013.