Numerical prediction for fire resistance of RC beams

Szymon Seręga[1] **and Adam Wosatko**[2]

[1,2] *Faculty of Civil Engineering, Cracow University of Technology, Warszawska 24, 31-155 Cracow, Poland*
e-mail: szymon.serega@pk.edu.pl [1], *awosatko@L5.pk.edu.pl* [2]

Abstract

Fire resistance of different structures is an important issue of their durability. Simple but effective tool to investigate multi-span reinforced concrete beams in fire conditions is discussed below. Essential assumptions for the theory and numerical algorithm are briefly reviewed. Two steps for nonlinear finite element analysis and two levels of observation are distinguished. The first step is the solution of nonstationary heat transfer in representative two-dimensional reinforced concrete cross-section of a beam exposed to fire. In the second step a nonlinear mechanical problem is calculated for the whole beam, i.e. the part with constant mechanical load and the part with an additional time-dependent thermal load due to fire. Global changes of curvature and bending moment functions imply degradation of the stiffness. Two benchmarks are briefly presented to confirm the correctness of the model.

Keywords: fire resistance, reinforced concrete, beams, thermal-mechanical analysis, finite element method

1. General assumptions and theory

Many different advanced approaches to thermo-mechanical analysis of reinforced concrete (RC) structures exist in the literature. One of the simplest, but effective application of nonlinear finite element (FE) computations for multi-span RC beams subjected to fire loading is concisely discussed below.

The first assumption is that the model of the beam is reduced to the neutral axis, so one-dimensional FE discretization is adopted. Additionally, at each node of FE a corresponding perpendicular cross-section is considered. In this cross-section, where thermal analysis is performed, two-dimensional FE discretization is introduced. Two levels of observation in the model coincide with two solution steps of the problem. Similar approach is employed in [2]. Details of the finite element method can be found e.g. in [1]. The problem is uncoupled, i.e. fire degrades the stiffness of the beam, but the effect of cracking does not change the thermal properties of RC. Transient heat transfer is solved for the cross-section of the heated part of the RC beam. The influence of moisture on thermal fields as well as resulting pore pressure are neglected in the analysis. The heat balance (Fourier-Kirchhoff) equation together with initial and boundary conditions, where heat flux is described by convection and radiation, are taken into account. The continuous multi-span beam has the possibility of free thermal elongation, therefore the resultant normal force equals zero. The Bernoulli hypothesis is applied, i.e. plane cross-sections remain plane after deformation. The tension stiffening between neighbouring cracks is skipped. Shear stiffness degradation and consequently possible shear failure are not considered in the model. The influence of self-balanced stresses due to the temperature gradient, that are normal to the longitudinal axis of the beam, is not included in the analysis. The free thermal strain field depends on the known temperature distribution in the cross-section. The uniaxial stress-strain relationships for concrete and steel are adopted according to the formulae defined in Eurocode 2-1-2. The balance of resultant forces is computed for each node of the beam and each time instant t. The current local equilibrium at nodes activates the stress state of the beam, while deflection function $w(t)$ is computed. Afterwards, bending moment $M(t)$ and total curvature $\kappa(t)$ are established. Free thermal curvature $\kappa_{th}(t)$ comes from heat transfer analysis. These variables update reduced stiffness $B(t)$:

$$B(t) = M(t) / \left[\kappa(t) - \kappa_{th}(t)\right] \tag{1}$$

which is the start point to find the solution for next time instant.

2. Numerical aspects of analysis

Both the thermal and mechanical problems are investigated by incremental-iterative routine for each time instant. The fire analysis is performed only for the concrete cross-section. The temperatures for reinforcing bars are determined from moving weighted least square (MWLS) approximation, cf. e.g. [4]. Figure 1(a) illustrates the regions where MWLS method is used.

After the first step, when the thermal problem is computed, the mechanical analysis is run for the same time instants. The incremental-iterative procedure is employed two times, first to find equilibrium at each node and second to examine global equilibrium. For this purpose the Newton-Raphson method is applied for the following system of equations:

$$\begin{bmatrix} \dfrac{\partial\, \mathbf{f}_{\mathbf{int}}^{N}}{\partial\, \varepsilon_0} & \dfrac{\partial\, \mathbf{f}_{\mathbf{int}}^{N}}{\partial\, \kappa} \\ \dfrac{\partial\, \mathbf{f}_{\mathbf{int}}^{M}}{\partial\, \varepsilon_0} & \dfrac{\partial\, \mathbf{f}_{\mathbf{int}}^{M}}{\partial\, \kappa} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_0 \\ \Delta\kappa \end{bmatrix} = \mathbf{r} \tag{2}$$

where the matrix operator consists of the corresponding derivatives of the resultant internal force vector $\mathbf{f}_{\mathbf{int}}$, $\Delta\varepsilon_0$ is the increment of strain at the cross-section centre of gravity, $\Delta\kappa$ is the increment of curvature of the beam, $\mathbf{r}$ is the vector of residual cross-section forces. The norm of $\mathbf{r}$ has to be small enough and then the new values of the stiffness $B(t)$ are known.

The nonstationary heat transfer problem is computed using the DIANA package. The second step is implemented and verified in a MATLAB code. This tool is a component of a future full program for fire analysis in RC beams.

3. Numerical examples

The first example is compatible with the experiment [3], where a one-span simply supported beam was tested. The following geometry: $L = 6.5$ m, $h = 0.6$ m, $b = 0.2$ m and uniform load $q = 10$ kN/m are introduced. A fire scenario is assumed for the beam according to ISO 834 from 3 sides, see also Fig. 1(c). Suitable properties and thermal constitutive relations are given for siliceous aggregate concrete and hot rolled steel in agreement with Eurocode 2-1-2. Selected results are shown in Fig. 1. A sudden increase of the deflection function is identified as total failure of the beam, see Fig. 1(b).

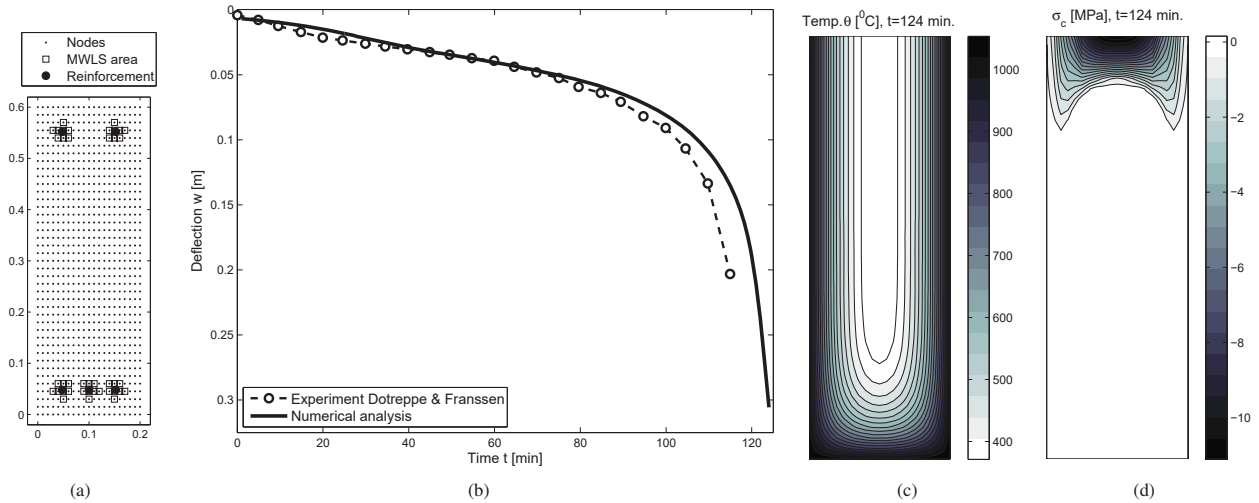

Figure 1: Plots for one-span beam: (a) – cross section with location of reinforcement and nodes for MWLS approximation; (b) – deflection-time diagram for span centre, cf. [3]; distribution of temperature θ (c) and stress σ_c (d) in central cross section before beam failure

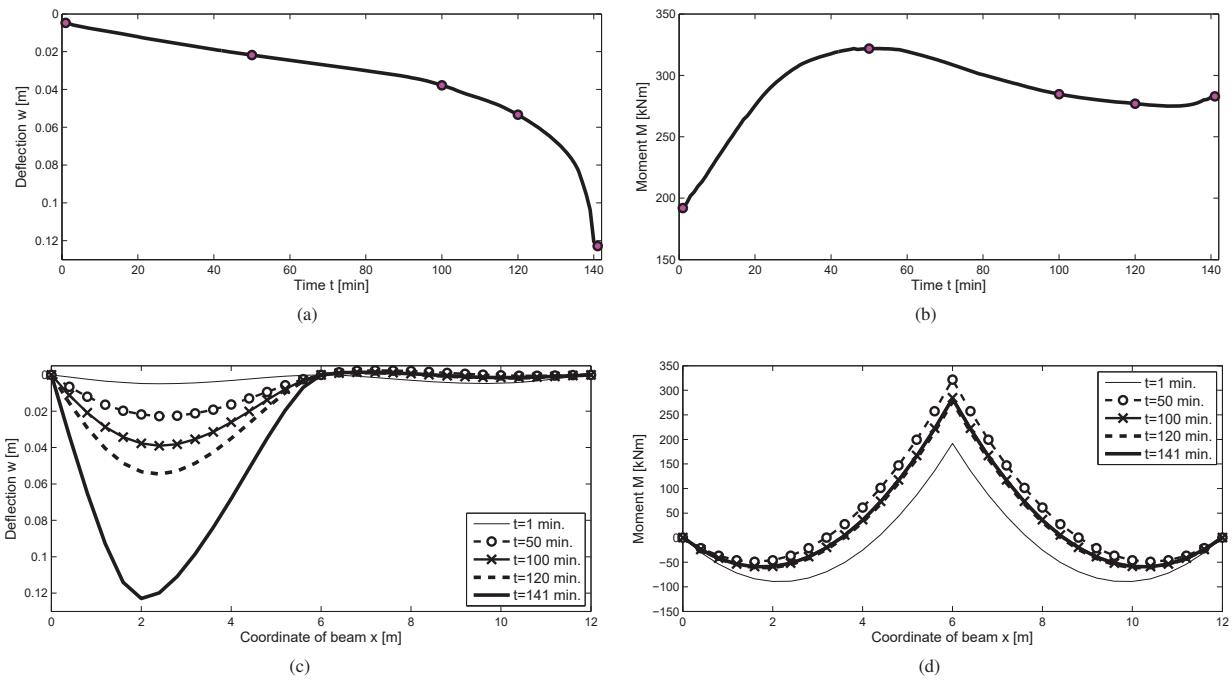

Figure 2: Plots for two-span beam exposed to fire in the left span: (a) – deflection-time diagram for centre of the left span; (b) – bending moment-time for middle support; functions of deflection (c) and bending moment (d) for selected time instants

The second test is performed for a two-span simply supported beam, where the length of one span is 6 m. The height of the cross-section does not change, but the width is now $b = 0.3$ m. The whole beam is uniformly loaded by $q = 38.3$ kN/m. This value is calculated as for accidental fire situation in accordance with PN-EN 1990 code. It should be emphasized that in this simulation fire heat acts only in the left span of the beam, see the deflection function in Fig. 2(c). It is noticed that the character of bending moment-time diagram in Fig. 2(b) is similar to that presented in [5].

References

[1] Cook, R. D., Malkus, D. S., Plesha, M. E. and Witt, R. J. *Concepts and Applications of Finite Element Analysis.* John Wiley & Sons, Hoboken, New Jersey, 4th edition, 2002.

[2] Di Capua, D. and Mari, A. R., Nonlinear analysis of reinforced concrete cross-sections exposed to fire. *Fire Safety Journal*, 42:139–149, 2007.

[3] Dotreppe, J.-C. and Franssen, J.-M., The use of numerical models for the fire analysis of reinforced concrete and composite structures. *Engineering Analysis*, 2(2):67–74, 1985.

[4] Milewski, S., Meshless finite difference method with higher order approximation – applications in mechanics. *Archives of Computational Methods in Engineering*, 19(1):1–49, 2012.

[5] Sadaoui, A. and Khennane, A., Effect of transient creep on behavior of reinforced concrete beams in fire. *ACI Materials Journal*, 109(6):607–616, 2012.