Self-stress control of real civil engineering tensegrity structures

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Abstract

The paper introduces the impact of the self-stress level on the behaviour of the tensegrity truss structures. Displacements for real civil engineering tensegrity structures are analysed. The analysis is performed by the second and third order theory. *Mathematica* software and *Sofistik* programme is applied to the analysis.

Keywords: truss, tensegrity, self-stress, control, non-linear theory

1. Introduction

Tensegrities are lightweight structures whose integrity is based on a balance between tension and compression. The concept of such structures concerns on specific trusses which consist of compression and tension components which stabilize each other despite the fact that there are mechanisms. Tensegrity as a structural system offers many advantages over conventional structural systems. Their main benefit is that under the right actuation they can maintain their stiffness during deployment without requiring external members [5].

The tensegrity concept has found applications in civil engineering. The first civil structure inspired to the tensegrity principle is the cable dome proposed by Geiger and first employed for the roofs of the Olympic Gymnastics Hall and the Fencing Hall in Seoul. An important example of tensegrity being employed in roof structures is the stadia at La Plata. The largest existing cable dome is the Georgia Dome designed for the Atlanta Olympics in 1996 [3]. Moreover double-layer tensegrity grids and foldable tensegrity systems has been in the development. The design of double grid systems has resulted in an interest in the application of tensegrity to bridge construction. A recent achievement in this regard is the Kurilpa Bridge in Brisbane, Australia. It is the world's largest "tensegrity-like" bridge, which was opened on the 4th of October 2009 [6]. The idea of tensegrity is also applied to build towers. The highest tensegrity tower (Warnow Tower) was built at the 2003 for the opening of the International Garden Exhibition in Rostock, Germany. The tower was designed by Mike Schlaich [1,4] and it is an example of "pure tensegrity" structure. The structure consists of six Simplex trusses of 8.3 m height, each made of three steel strings in compression (Ø=273 mm, t=12 to 40 mm) which are stabilized by three diagonal cables (Ø=50 and 75 mm) and three horizontal cables (Ø=30 and 50 mm). Warnow Tower, measuring 49.2 meters tall and, with the addition of a 12.5 meter "needle", totaling 62.3 meters in height.

The stiffness of tensegrities comes from topology, configuration, pre-stress and initial axial element stiffness. An important benefit of tensegrity structures is possibility to control a stiffness due to existing infinitesimal mechanisms. The stiffness can be increased by self-stress forces added in truss members and also by applied external forces.

The objective of the present paper is t[analyse a possibility to control properties of real tensegrity structures e.g. the stiffness by self-stress state.

2. Mathematical model of tensegrity structures

Identification infinitesimal mechanisms and a self-stresses in structures is possible by using the second order theory:

$$\left(\mathbf{K}_L + \mathbf{K}_{NL}^{I}\right)\mathbf{q} = \mathbf{P}\,, \tag{1}$$

where $\mathbf{q}$ is displacement vector, $\mathbf{P}$ is load vector, $\mathbf{K}_L$ is linear stiffness matrix and $\mathbf{K}_{NL}^{I}$ is pre-stress stiffness matrix called the geometric stiffness matrix. To calculate the pre-stress stiffness matrix a self-stress, based upon a singular value decomposition of the linear stiffness matrix $\mathbf{K}_L$ [2], is defined.

The second order theory (1) do not include the influence of external loads on the stiffness of the structure. These loads cause displacements following the infinitesimal mechanism and lead to more compression of the structure by introducing additional tensile and pressure forces in elements. In order to take account the effect of additional compression geometrically nonlinear model (the third order theory) should be used:

$$\left(\mathbf{K}_L + \mathbf{K}_{NL}^{I} + \mathbf{K}_{NL}^{II}\right)\mathbf{q} = \mathbf{P} \tag{2}$$

where $\mathbf{K}_{NL}^{II}$ is the initial strain matrix.

The equation (2) is non-linear and an incremental resolution method (for example the Newton–Raphson method) is required in association with an iterative process.

3. Examples

The impact of the level of self-stress on displacements for Warnow Tower (Fig. 1) is studied in this paper. Concentrated load forces P_i=-1 kN are considered and displacements loaded nodes in the direction of the external forces are determined. Values of the self-stress are limited by load capacity of struts. Analysis of three models is presented in the paper. The first model consists of one Simplex truss, second – two Simplex trusses and third – six Simplex trusses. The last example as well as more real civil engineering structures will be presented on conference. In the first case calculations are made in *Mathematica* environment. In the second case calculations are made using commercial program *Sofistik* that served to accomplish the fully geometrical non-linear analysis.

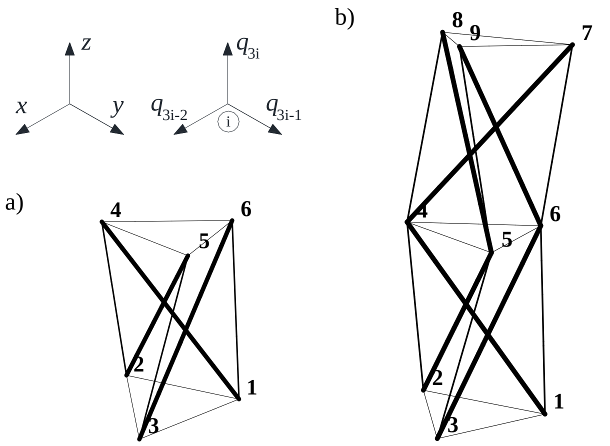


Figure 1: Scheme of models: a) first model, b) second model

The following mechanical and geometric properties are chosen for the analysis:

– steal S460N, Young's modulus E = 210GPa,
– diagonal cables: Ø 75 mm, I=1.553·10^{-6} m^4, A=4.418 10^{-3} m^2, L=6 m, load capacity N_{Rd} = 3735 kN, self-stress 0.338499·S,
– horizontal cables: Ø 50 mm, I= 3.068·10^{-7} m^4, A=1.963 10^{-3} m^2, L=10.66 m, load capacity N_{Rd} = 1209 kN, self-stress: 0.138078·S,
– struts: Ø 273 mm with thickness 40 mm, I=2.046·10^{-4} m^4, A=2.928·10^{-2} m^2, L=10.66 m, load capacity $N_{b,R,d}$ = -3735 kN, self-stress -0.424964·S.

For the first model (Fig. 1a) dependence of factor of self-stress S on displacement q_{12} is presented in Fig. 2. For the second model (Fig. 1b) two independent factors of self-stress are considered: S_1 – on the lower level of the structure and S_2 – on the higher level. The influence of both self-stresses on the q_{21} displacement are presented in Fig. 3 with the use of second and third order theory.

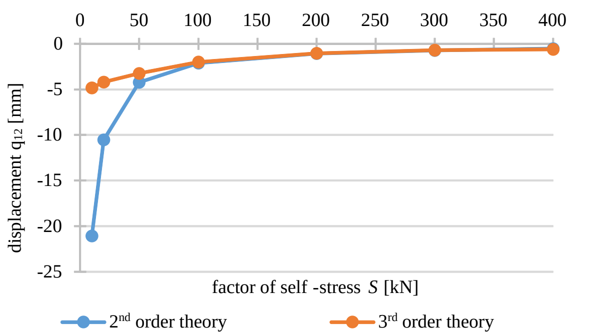
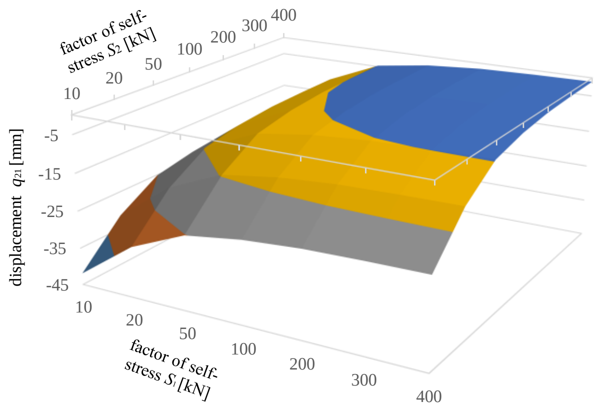


Figure 2: Values of displacement q_{12} for the first model (external load: P_{12}=-1 kN)

4. Conclusions

The analysis of real civil engineering tensegrity structures shows the possibility to control displacements by adjusting prestressing forces. For multi-module structures the control is multi-parameter by analysing separate self-stresses. Nodal displacements decrease significantly with increase of internal forces of self-stress as well as of the influence of geometrical nonlinearity is clearly seen.

a)



b)

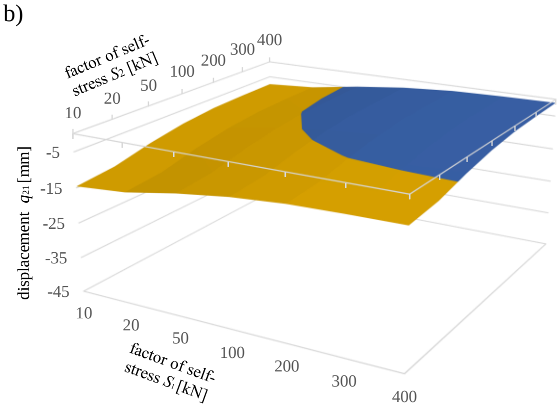


Figure 3: Displacement q_{21}: a) 2nd order, b) 3rd order theory

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