Probabilistic homogenization of the random HDPU composite with ellipsoidal carbon black particle reinforcement by the Iterative Stochastic Finite Element Method

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Abstract

The main aim is determination of basic probabilistic characteristics of the orthotropic homogenized elastic properties of the periodic high density polyurethane (HDPU) polymer reinforced with Carbon Black (CB) particles having ellipsoidal shape. Homogenization problem, solved by the Iterative Stochastic Finite Element Method (ISFEM) is implemented according to the stochastic perturbation, Monte Carlo simulation and semi-analytical techniques with the use of cubic Representative Volume Element (RVE) of this polymer containing single particle. The given input Gaussian random variable is Young modulus of the HDPU matrix, while 3D homogenization scheme is based on numerical determination of the strain energy of the composite RVE under nine uniform unitary stretches carried out in the FEM system ABAQUS. The entire series of several deterministic solutions with varying HDPU Young modulus serve for the Least Squares Method (LSM) recovery of polynomial response functions finally used in stochastic Taylor expansions inherent for the ISFEM. It is numerically investigated (1) if the resulting homogenized characteristics are also Gaussian and (2) how the uncertainty in matrix Young modulus affects the effective stiffness tensor components.

Keywords: homogenization, Stochastic Finite Element Method, particle reinforced composite, Monte-Carlo simulation, semi-analytical probabilistic technique

1. Introduction

Since many years homogenization technique has been widely used for determination of properties of various composite materials. Current research efforts in this field concerning particle-reinforced composites stiffness tensor are either purely focused on analytical considerations [1], their computational validation [2], automatic meshing generation [3] or, alternatively, on its probabilistic characteristics when their material, geometry or spatial distribution of particles is random [4]. This work fits well into these research areas and concern the random stiffness tensor of homogenized HDPU composite with ellipsoidal CB particles.

2. Theoretical background and FEM model

Let us consider a periodic composite with High Density Polyurethane (HDPU) matrix reinforced with ellipsoidal particles of Carbon Black (CB). Homogenized tensor associated with the Representative Volume Element of such a composite is orthotropic with 9 independent components usually represented according to the Voigt notation in the following form:

$$C_{ij}^{eff} = \begin{bmatrix} C_{11}^{eff} & C_{12}^{eff} & C_{13}^{eff} & 0 & 0 & 0 \\ & C_{22}^{eff} & C_{23}^{eff} & 0 & 0 & 0 \\ & & C_{33}^{eff} & 0 & 0 & 0 \\ & & & C_{44}^{eff} & 0 & 0 \\ & sym. & & & C_{55}^{eff} & 0 \\ & & & & & C_{66}^{eff} \end{bmatrix}. \quad (1)$$

This tensor is further reduced to 6 independent components when two axes of the reinforcing particle have the same length - $C_{11} = C_{33}$, $C_{44} = C_{66}$ and $C_{12} = C_{23}$. Such a composite is modelled here in the FEM system ABAQUS with the CB axes of 0.2 (axis x and z) and 0.25 (axis y, Fig. 1b) and computed for the strain energies of the unit uniaxial, biaxial and shearing stretches (Fig. 2).

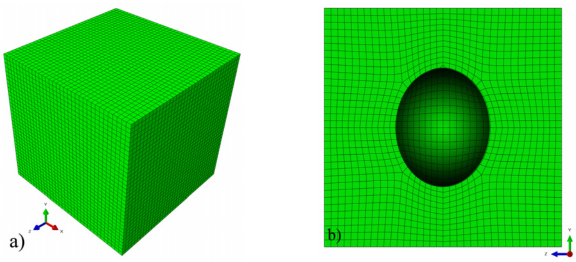

Figure 1: FEM mesh (a) and cross-section (b) of the RVE.

The mesh of this composite (Fig. 1a) is made of over 50 000 20-noded brick finite elements with a second-order stress approximation and a series of computations is made in the elastic regime to calculate total strain elastic energies in close neighbourhood ($\pm 5\%$) of the expected value of HDPU Young modulus $E = 4\ MPa$ and Poisson ratio $\nu = 0.34$. This series of 11 solutions is further used to build effective stiffness tensor representation and each of its 6 components C_{ij}^{eff} is approximated via the Least Squares Method (LSM) using the additional polynomial basis. This is done to retrieve the polynomial representation of the C^{eff} with respect to the HDPU Young modulus, which, in turn, serve as the basis for

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computation of the first four probabilistic characteristics of the effective stiffness tensor, i.e. expectation, coefficient of variation, skewness and kurtosis – all as functions of the coefficient of random dispersion for the input random parameter (HDPU Young modulus). Probabilistic method engaged to this computation is triple – contains the iterative stochastic perturbation technique (SPT) of the optimized (4th) order [5], the Monte Carlo simulation (MCS) with 250 000 trials as well as the semi-analytical technique implemented in the symbolic computer program consisting in direct determination of probabilistic characteristics from their integral (AM).

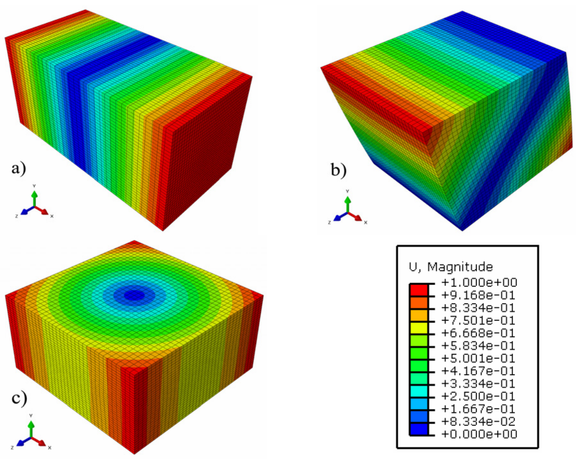

Figure 2: Deformation maps for a) uniaxial, b) shear and c) biaxial unitary extensions of the RVE.

3. Computer simulation results

Final diagrams of probabilistic characteristics of the effective stiffness tensor C_{ii}^{eff} (Figs 3-6) present the expected value (Fig. 3), coefficient of variation (Fig. 4), skewness (Fig. 5) and kurtosis (Fig. 6) of this variable in function of the input coefficient of variation of the Young modulus α.

Expected value and coefficient of variation are both increasing with the increasing input coefficient of variation α and correlation in-between the three probabilistic techniques is here very high. The expected values of both C_{11}^{eff} and C_{33}^{eff} are here exactly identical (only two extensional components are independent in such a composite). Interestingly, the output uncertainty is much larger than the input one. This means, that small variations of the matrix Young modulus cause much larger fluctuation of the effective stiffness tensor components, which is also well depicted on Fig. 3, where the expected values of C_{ii}^{eff} sharply increase with increasing input parameter α.

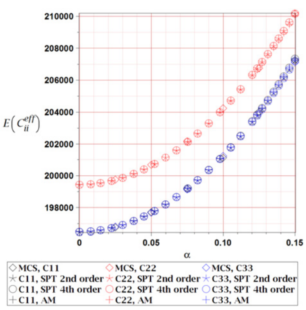

Figure 3: Expected value of the effective tensor components.

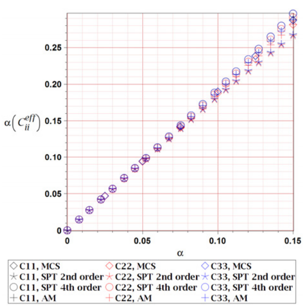

Figure 4: Coefficient of variation of the effective tensor components.

Higher order probabilistic characteristics, i.e. skewness and kurtosis are all positive and increase together with the input uncertainty. A coincidence in-between these three probabilistic methods is no longer perfect, but still they are smooth and have no local discontinuities. The SPT slightly diverges from $\alpha = 0.08$ for all three tensor components and both probabilistic characteristics.

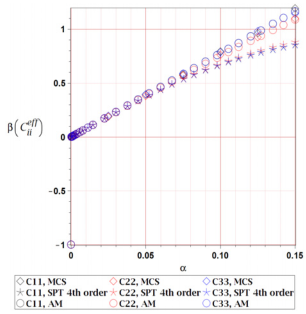

Figure 5: Skewness of the effective tensor components.

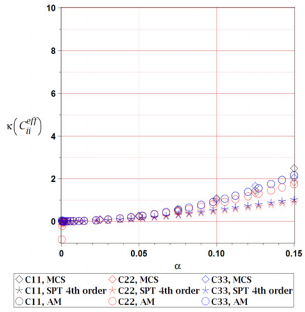

Figure 6: Kurtosis of the effective tensor components.

4. Conclusions

This work presents an application of the Iterative Stochastic Finite Element Method in determination of the homogenized orthotropic effective stiffness tensor $\boldsymbol{C}^{eff}$ of HDPU composite reinforced with CB particles of ellipsoidal shape. It proves, that the $\boldsymbol{C}^{eff}$ is sensitive to variation of the matrix mechanical properties, that this tensor is not Gaussian for Gaussian input random parameter of matrix Young modulus and, additionally, confirms that such a homogenized composite has an orthotropic stiffness tensor.

References

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