Influence of elastic supports on non-linear steady-state vibrations of Zener material plates

Przemysław Litewka* and Roman Lewandowski*

Institute of Structural Engineering, Poznan University of Technology
ul. Piotrowo 5, 60-965 Poznań, Poland
e-mail: przemyslaw.litewka@gmail.com

Abstract

This paper is devoted to numerical testing of influence of elastic supports on forced harmonic vibrations of von Kármán geometrically non-linear plates, made of the Zener viscoelastic material. The dynamic problem is formulated in the frequency domain, using a consistent harmonic form of the solution for plate displacements and the time-averaged principle of virtual work. The amplitude equation is derived from the harmonic balance method. Then, the plate is discretized with 8-noded rectangular plate finite elements with selective-reduced integration. Numerical examples of one- and two-span plates with elastic supports are solved to find response curves with the use of a path-following method. Some types of unusual dynamic behaviour are found, which occur due to the combination of geometric non-linearity, the adopted model of viscoelasticity and the presence of elastic supports.

Keywords: plate vibrations, von Karman non-linearity, Zener material, harmonic balance method, elastic supports, response curves

1. Introduction

Reduction of excessive and undesired vibrations of structures or their elements is an important issue in modern technology. Some of methods used to achieve this goal involve special materials exhibiting damping properties, which can be represented by different viscoelastic models.

In the analyses Kelvin-Voigt approximation is used usually, e.g. [1]. The Golla-Hughes-McTavish material was adopted in [2]. The Zener material model was used by the authors in [3]. Here that model and the method of solution of geometrically non-linear harmonically forced vibrations of plates was used to analyze the influence of elastic supports. Plates with physically linear and non-linear Winkler-type supports were considered. Cases of unusual dynamic behaviour resulting from a complex interplay of geometric non-linearity, character of the Zener material and the presence of elastic supports were found and discussed.

The paper sketches the way of derivation of the amplitude equation in Section 2. Section 3 gives some information on the finite element discretisation used, while Section 4 presents results of numerical analysis and a discussion.

2. Derivation of amplitude equation

The functions of displacements of the mid-plane of a plate present in the model are: in-plane translations u^0, v^0; deflection w^0 and angles of rotation φ_x and φ_y. The vectors of von Kármán strains, rotational, shear and in-plane, respectively, are

$$\boldsymbol{\varepsilon}_r = \{\varphi_{x,x} \;\; \varphi_{y,y} \;\; (\varphi_{x,y} + \varphi_{y,x})\}^T, \; \boldsymbol{\varepsilon}_s = \{(w^0_{,x} + \varphi_x) \;\; (w^0_{,y} + \varphi_y)\}^T,$$

$$\boldsymbol{\varepsilon}_p = \{u^0_{,x} \;\; v^0_{,y} \;\; (u^0_{,y} + v^0_{,x})\}^T + \tfrac{1}{2}\{(w^0_{,x})^2 \;\; (w^0_{,y})^2 \;\; 2w^0_{,x}w^0_{,y}\}^T \tag{1}$$

The internal forces: bending and torsional moments; two shear forces and in-plane forces, respectively, are assembled as

$$\mathbf{M} = \{M_x \;\; M_y \;\; M_{xy}\}^T, \; \mathbf{Q} = \{Q_x \;\; Q_y\}^T, \mathbf{N} = \{N_x \;\; N_y \;\; N_{xy}\}^T \tag{2}$$

The physical relations for the Zener viscoelasticity have the form

$$\begin{aligned} \mathbf{M} + \tau\dot{\mathbf{M}} &= \mathbf{D}_0\boldsymbol{\varepsilon}_r + \tau\mathbf{D}_\infty\dot{\boldsymbol{\varepsilon}}_r \\ \mathbf{Q} + \tau\dot{\mathbf{Q}} &= \mathbf{A}_{q0}\boldsymbol{\varepsilon}_s + \tau\mathbf{A}_{q\infty}\dot{\boldsymbol{\varepsilon}}_s \\ \mathbf{N} + \tau\dot{\mathbf{N}} &= \mathbf{A}_0\boldsymbol{\varepsilon}_l + \tau\mathbf{A}_\infty\dot{\boldsymbol{\varepsilon}}_l \end{aligned} \tag{3}$$

where the classical stiffness matrices $\mathbf{D}_0$ and $\mathbf{D}_\infty$ for bending, $\mathbf{A}_0$ and $\mathbf{A}_\infty$ for extension and $\mathbf{A}_{q0}$ and $\mathbf{A}_{q\infty}$ for shear at initial and infinite time state are present.

Harmonic vibrations excited by a transverse loading

$$\mathbf{p}_w(x,y,t) = \mathbf{p}_{wc}(x,y)\cos\lambda t + \mathbf{p}_{ws}(x,y)\sin\lambda t \tag{4}$$

with the frequency λ are analysed. The plate response compatible with the geometric non-linearity is assumed as

$$\begin{aligned} \mathbf{q}_w(x,y,t) &= \{w^0\} = \mathbf{q}_{wc}(x,y)\cos\lambda t + \mathbf{q}_{ws}(x,y)\sin\lambda t \\ \mathbf{q}_r(x,y,t) &= \begin{Bmatrix}\varphi_x \\ \varphi_y\end{Bmatrix} = \mathbf{q}_{rc}(x,y)\cos\lambda t + \mathbf{q}_{rs}(x,y)\sin\lambda t \\ \mathbf{q}_t(x,y,t) &= \begin{Bmatrix}u^0 \\ v^0\end{Bmatrix} = \mathbf{q}_{tc}(x,y)\cos^2\lambda t + \mathbf{q}_{t0}(x,y)\cos\lambda t\sin\lambda t + \\ &+ \mathbf{q}_{ts}(x,y)\sin^2\lambda t \end{aligned} \tag{5}$$

The time-averaged expression of the virtual work with the harmonic balance method, applied to solve the physical relations in Eq (3), yields the non-linear amplitude equation. For the details of this derivation, see [3].

3. Finite element discretization

The amplitude equation is discretized using the finite element methodology. Here the 8-noded rectangular elements with selective reduced integration for the shear terms and bi-quadratic shape functions are applied. The discretized amplitude equation with the unknown nodal amplitudes $\mathbf{q}_e$ can be given as

* The research reported in this paper was financed by the projects: 2013/09/B/ST8/01733 from the National Science Centre and the internal University grant 01/11/DSPB/0800.

$$\delta\mathbf{q}_e{}^T\mathbf{P}_e + \delta\mathbf{q}_e{}^T\left(-\lambda^2\right)\mathbf{M}_e\mathbf{q}_e + \delta\mathbf{q}_e{}^T\mathbf{K}_{el}\mathbf{q}_e + \\ + \delta\mathbf{q}_e{}^T\left[\mathbf{K}_{en}\left(\mathbf{q}_e\right)\right]\mathbf{q}_e = 0 \tag{6}$$

with the vector of nodal load amplitudes $\mathbf{P}_e$, the mass matrix $\mathbf{M}_e$ as well as the linear and non-linear stiffness matrices $\mathbf{K}_{el}$, $\mathbf{K}_{en}$. for the plate. The latter ones include terms due to elastic supports. In order to solve this equation and find the response curves the arc-lenght method is applied. It also involves a computaton of the consistent linearization of the non-linear stiffness matrix $\mathbf{K}_{en}$. The deatils are given in [3].

4. Numerical analyses and discussion

In this section two numerical examples of plates with various support conditions are solved. The point load in the form of p_{wc} amplitude, see Eq (4), is applied at the point A and the response in the form of the total deflection amplitude

$$q_w = \sqrt{q_{wc}{}^2 + q_{ws}{}^2} \tag{7}$$

computed from sine and cosine amplitudes measured at the same point A is considered.

The elastic supports in the model are of Winkler non-linear type with the reaction-deflection (R_w–q_w) relation given by

$$R_w = k_1 q_w + k_3 q_w^3 \tag{8}$$

The plates have the thickness $h = 0.1$ m and are made from the viscoelastic material with $E_0 = 7\cdot 10^6$ N/m², $E_\infty = 10^7$ N/m², $\tau = 0.0175$ s, $\nu = 0.3$ and $\rho = 1250$ kg/m³. In all the cases there are 6×6 finite elements per one plate span.

First, let us consider a single-span square plate shown in Fig. 1, simply supported at one pair of opposite edges and elastically supported at the other edges. The excitation amplitude is $p_{wc} = 1500$ N. The response curves for varying value of the linear support stiffness k_1 (with $k_3 = 0$) are shown in Fig. 2. It is interesting to observe, that the peak amplitude values in these curves related to the first resonance frequency are not a monotonic function of support stiffness. Such a behaviour was observed in [3] for the relation of the peak value and the relaxation time τ. This property is attributed to the nature of Zener model of viscoelasticity. There exists a critical combination of model parameters which yields the extreme peak value. Here the critical support stiffness corresponding to the material data given above is about $k_1 = 6\cdot 10^3$ N/m.

For this value the influence of non-linear component in Eq (8) is further investigated and the response curves are shown in Fig. 3. One can notice, that the increase of the positive coefficient k_3 leads to overstiffening of the system, manifested in a more inclined response curve at the resonance peak, while the negative k_3 has an opposite effect.

Then a two-span plate with the elastic intermediate support, see Fig. 1, with the excitation amplitude $p_{wc} = 2500$ N is considered. The influence of the linear stiffness coefficient k_1 on response curves is presented in Fig. 4. Here an interesting behaviour is visible, too. Changes in sequence of vibration modes for varying k_1 lead to unusual results, especially the looped curve for $k_1 = 10^2$ N/m is worthy mentioning. Apparently, it corresponds to the situation when two first modes of vibration have similar resonance frequencies and do interact.

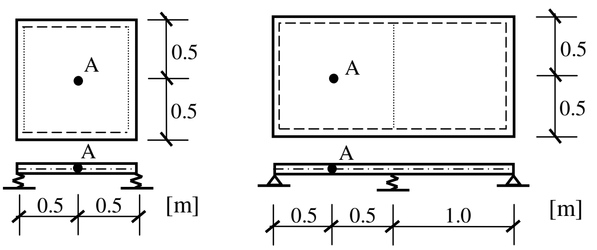

Figure 1: Analyzed plates

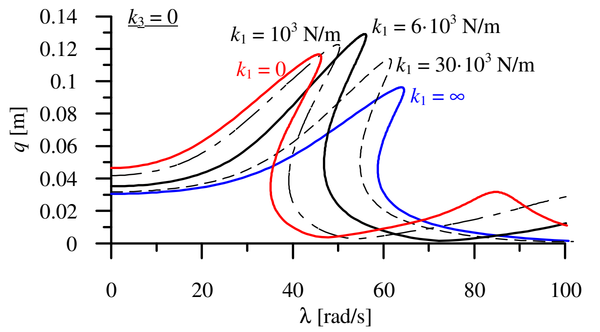

Figure 2: Response curves for the single-span plate with $k_3 = 0$

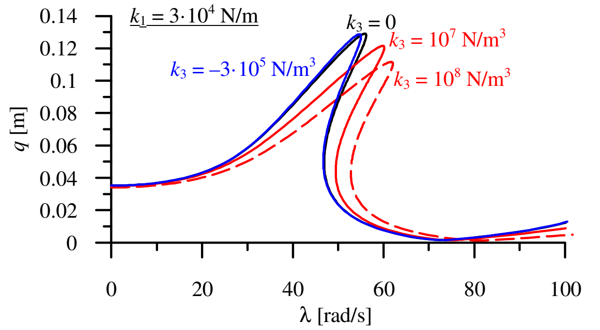

Figure 3: Response curves for the single-span plate with $k_1 = 3\cdot 10^4$ N/m

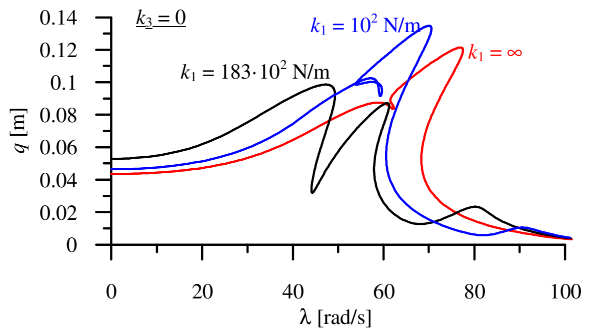

Figure 4: Response curves for the two-span plate with $k_3 = 0$

References

[1] Amabili, M., Nonlinear vibrations of viscoelastic rectangular plates, *J Sound and Vibration*, 362, pp. 142-156, 2016.

[2] Kumar, R.S., Ray, M.C., Smart damping of geometrically nonlinear vibrations of functionally graded sandwich plates using 1-3 piezoelectric composites, *Mech Advanced Mater and Struct*, 23(6), pp. 652-669, 2016.

[3] Litewka, P., Lewandowski, R., Non-linear harmonically excited vibrations of plates with Zener material, *Nonlinear Dynam*, (submitted 2017).