

Dynamic properties of tensegrity structures

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Abstract

The present paper focuses on the analysis of dynamic properties of different tensegrity structures. Two aspects are analysed: free vibration of single tensegrity modules and behaviour of a tensegrity plate under dynamic load. Influence of self-stress on natural frequencies of selected modules is determined and a nodal displacement of a plate built from 3-strut simplex modules is analysed. Possibility of active control of tensegrity systems in terms of dynamics is verified.

Keywords: tensegrity, dynamic analysis, free vibration, eigenfrequencies, active control

1. Introduction

Tensegrities [4,5] can be defined as cable-strut structures with a special configuration of nodes and elements that form a statically indeterminate system in a stable equilibrium. They consist of a discontinuous set of compressed members inside a continuous set of tensioned elements with no compressive stiffness. Even simple tensegrity structures have a complicated geometry and some untypical, unique features that result from the infinitesimal mechanisms balanced with self-stress states.

Despite many studies concerning geometry, form-finding and statics of tensegrity systems, researchers have rarely focused on dynamics of these structures. Although several works on dynamic behaviour and vibration control of tensegrities have been written [2,3,6], few information about dynamic properties of tensegrity structures can be found in the literature.

The aim of this paper is the estimation of the influence of self-stress on dynamic properties of tensegrity structures and verification of the possibility of active control. Two aspects are analysed: natural frequencies of typical tensegrity modules and dynamic response of a tensegrity plate.

2. Dynamics of tensegrity trusses

From mechanical point of view, tensegrities can be regarded as truss structures and described using the equations from the mechanics of trusses. Discrete equations of motion for the undamped system have the following form:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{P}(t), \quad (1)$$

where: \mathbf{M} – mass matrix, \mathbf{K} – stiffness matrix, \mathbf{q} – vector of nodal displacements, \mathbf{P} – vector of nodal loads.

All the analyses performed for the purpose of this paper were carried out in accordance with a finite element method (FEM) formalism [1]. The calculations were conducted using a Newmark method, according to the 3rd order theory, which corresponds to a geometrically nonlinear analysis.

The stiffness matrix considered in the performed analyses consists of three elements: $\mathbf{K} = \mathbf{K}_L + \mathbf{K}_G + \mathbf{K}_{NL}$, where \mathbf{K}_L is

a linear stiffness matrix, \mathbf{K}_G allows to include self-stress in the calculations, \mathbf{K}_{NL} enables to take into account geometrical nonlinearities.

Two types of analyses were performed: analysis of free vibration of selected single tensegrity modules, analysis of a tensegrity plate under dynamic load.

Calculations were performed using a FEM software – SOFiSTiK (educational licence). All the analysed structures were supported. Strengths of structural members were included in the analysed numerical models.

3. Free vibration of tensegrity modules

Four basic tensegrity modules were analysed: a 4-strut simplex (S4), a 3-strut simplex (S3), an expanded octahedron (O) and a truncated tetrahedron (T) (Figure 1). All modules had geometry inscribed into a cube of edge length 1 m.

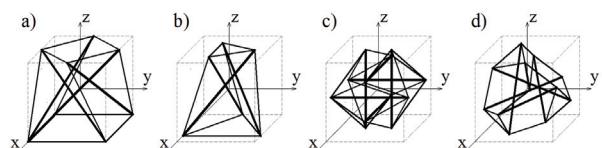


Figure 1: Analysed tensegrity modules: a) 4-strut simplex, b) 3-strut simplex, c) expanded octahedron, d) truncated tetrahedron.

Struts were modelled as tubes with outer diameter 76,1 mm and inner 70,3 mm made of steel S355J2, cables as bars with diameter 20 mm made of steel S460N.

Influence of self-stress on the first natural frequency of the analysed modules is shown in Figure 2. Other eigenfrequencies have also been considered, but due to the minimal influence of self-stress on their values the Authors have decided not to present those results. The parameter S_0 is a multiplier used to unify the values of prestressing forces. In the analysed examples the maximum force in a cable is $0,333333 \cdot S_0$ and in a strut - $0,421637 \cdot S_0$, which does not exceed the load-carrying capacity of these elements. The performed analysis show that the first natural frequency of each module equals zero when no prestressing forces are applied – this results from infinitesimal

mechanisms that occur in tensegrity systems. When the values of self-stress forces increase, the considered frequency grows, but the growth rate depends on the analysed module.

The biggest increase in the values of the first natural frequency can be observed for self-stress forces S_0 between 0 and approx. 10 kN. Above 10 kN the growth becomes slower – increasing S_0 from 10 kN to 300 kN (which is the maximum level safe for the structure) f_1 can be increased approx. 6 times.

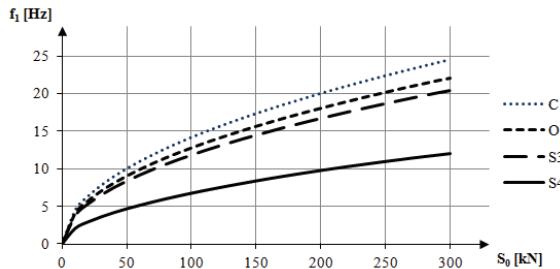


Figure 2: Influence of self-stress on the first natural frequency of four tensegrity modules

4. Tensegrity plate under dynamic load

A tensegrity plate constructed from 14 connected 3-strut simplex modules, with geometry inscribed into a cube of edge length 1 m, was analysed (Figure 3). The same cross-sections were applied as in the case of single modules described in the previous section.

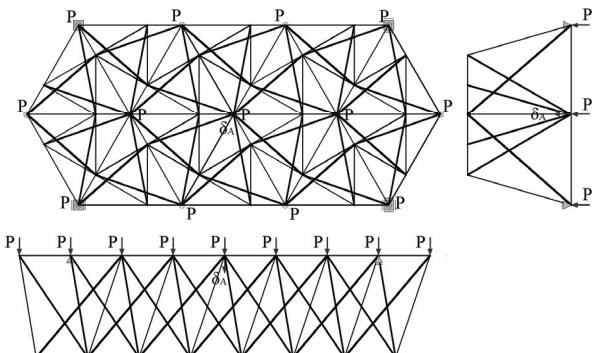


Figure 3: Tensegrity plate

The plate was loaded with concentrated forces applied to the upper nodes of the structure. The load value corresponds to the free fall of a mass 1 t from the height 30 m (without air resistance), which decelerates during the hit within 0,1 s. The function of each nodal force is shown in Figure 4.

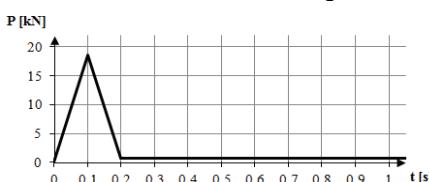


Figure 4: Nodal load function

The aim of the performed analysis was to investigate how the applied self-stress influences the vertical displacement of the central upper node – marked δ_A in Figure 3. The calculations were conducted according to the geometrically non-linear theory, using the Newmark method.

It can be observed that the displacement value decreases when the prestressing forces grow (Figure 5). In the analysed range of self-stress values, δ_A can be decreased twice – from

80 mm to about 40 mm. This is a good example of a potential structural control – properties of the structure can be changed by an adjustment of self-stress forces.

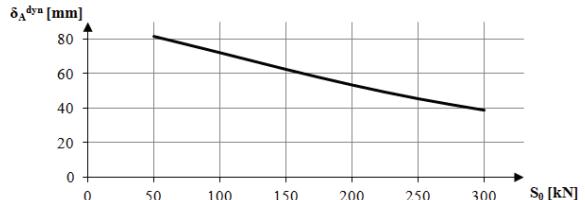


Figure 5: Influence of self-stress on vertical displacement of node A

Figure 6, on the other hand, shows how the dynamic factor depends on self-stress values. The dynamic factor was calculated for the static load of 10 kN, which is approximately a mean value of the applied dynamic load.

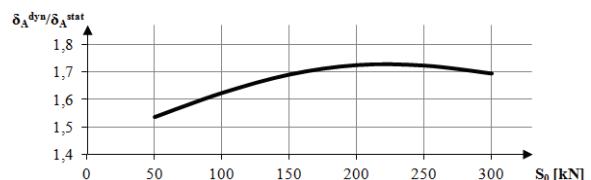


Figure 6: Influence of self-stress on the dynamic factor

5. Conclusions

The non-linear analyses performed for the purpose of this paper show that dynamic behaviour of tensegrity systems depends strongly on self-stress applied to these structures. By adjusting prestressing forces, the first natural frequency of tensegrities can be changed and a level of displacements of the structures subjected to dynamic loads can be controlled.

Self-stress influences both dynamic properties of tensegrity systems in terms of their free vibration and the global stiffness of the structures under dynamic load. The bigger the value of self-stress is, the stiffer the structure becomes and the smaller the nodal displacements are.

Further work will focus on the study of other types of tensegrity structures under various dynamic loads. Influence of self-stress on internal forces will be analysed.

References

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