# Non-classical effects of the transverse core compressibility on the static and dynamic response of soft core sandwich structures

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Abstract

The present contribution is concerned with the definition of higher-order models for sandwich plates and shells together with appropriate solution procedures for the numerical analysis of non-classical effects in static and dynamic instability problems of soft core sandwich structures. The structural model is based on a higher-order displacement expansion in an effective multilayer approach. Using a v. Kármán type approach, a geometrically nonlinear higher-order sandwich model is derived. A numerically efficient analytical solution to a class of static and dynamic instability problems is derived by means of an extended Galerkin procedure. As an alternative for more sophisticated problems, a finite element implementation is provided. The model is applied successfully to a variety of different static and dynamic instability problems, showing the necessity for including the transverse compressibility of the core for obtaining adequate solutions.

*Keywords: Sandwich structures, Higher-order theory, Static and dynamic instability, Local effects, Finite elements.*

## 1. Introduction

Structural sandwich panels consisting of two high-density face sheets bonded to a weak flexible core made of a soft material are important elements in many fields of modern lightweight construction (Vinson [5]). The main advantage is their high bending stiffness in conjunction with a rather low specific weight. Their main field of application is the aerospace field, the wind energy sector as well as all other fields of transport. Important examples are also found in the civil and naval engineering sectors. In most engineering analyses, sandwich panels are analysed in terms of classical of similar laminate theories or in terms of multilayer models. In most cases, the transverse flexibility of the core is neglected for simplicity. The present contribution is concerned with a higher-order model accounting for this effect and thus enabling a refined integrity assessment.

## 2. Higher order sandwich shell theory

In order to account for core flexibility effect, the present model utilizes an effective multilayer approach, treating all principal layers - top and bottom face sheets as well as the core - separately. For the face sheets, a classical Kirchhoff-Love model is used, whereas a second and first order power-series expansion is used for the core displacements (Hohe and Librescu [3]). Using the nonlinear Green-Lagrange tensor for representation of the strains together with Hamilton's principle and neglecting all transverse nonlinearities, a v. Kármàn type nonlinear model for shallow sandwich shells is derived.

## 3. Solution procedures

### *3.1. Extended Galerkin procedure*

In order to derive a numerically efficient approach, an extended Galerkin approach is employed. Using a double sine series expansion for both, the average transvers displacements of the face sheets as well as for the difference in their transverse displacement, a mathematical model for simply supported sandwich shells with rectangular projection is derived, where the modal amplitudes $w_{mn}^{a}$ of the overall stability and $w_{pq}^{d}$ of the face wrinkling instability are the only unknown variables. The cubic system is easily solved by Newton's method.

### *3.2. Finite element implementation*

As an alternative to the analytical solution strategy avoiding its restriction to simply supported shells with rectangular projection, the higher-order sandwich model from Sec. 2 has been implemented into a finite element program by Demiray et al. [1]. This approach uses a discrete Kirchhoff approach for the face sheets together with a transversely linear and quadratic shape function for the core. Based on these assumptions, a three-node triangular element with six degrees of freedom per node is derived.

## 4. Examples

### *4.1. Time dependent response under rapid loading*

As a first example, the transient response of a plane sandwich plate loaded by rapidly applied in-plane deflection (linear increase over 5 ms) is considered. The resulting edge load $N_{11}^{a}$ is presented in Figure 1. For comparison, the problem has also been analysed by a classical incompressible core sandwich shell model. It is observed that due to the formation of a face wrinkling instability in addition to the overall instability, a distinctively weaker response with a lower frequency of the emerging free vibration is obtained (Hohe et al. [4]).

### *4.2. Load-frequency interaction*

To investigate the effect of static preloads on the natural frequencies in more detail, the problem is transformed into an eigenvalue problem by decomposing the modal amplitudes into a large static one and a small oscillating part (Hohe [2]). In Figure 2, the first natural frequency is presented as a function of

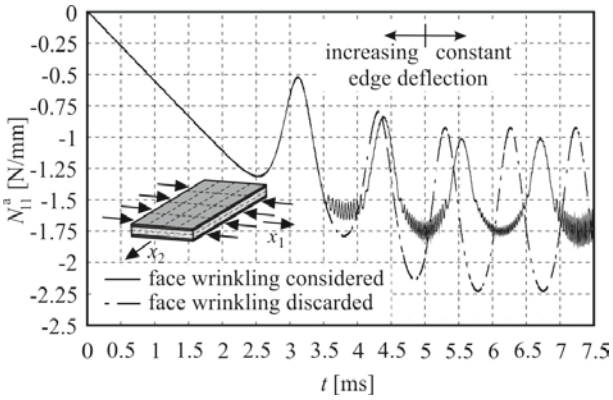


Figure 1: Transient response under rapidly applied load.

the applied edge deflection for different initial geometric imperfections. Strong effects are obtained when the resulting edge load is close to the global and local buckling loads.

*4.3. Two-dimensional problems*

As a more sophisticated example, a simply supported sandwich plate with central hole under in-plane compression is analysed by the finite element implementation of the sandwich model. Due to the stress concentration caused by the hole, a face wrinkling instability emerges from the notch roots as shown in Figure 3. The finite element model is able to accurately predict this interaction of the local instability with structural features.

*4.4. Localized instabilities*

As a final example, a single edge clamped sandwich beam loaded by a prescribed rotation φ of the free end is considered. The problem is again solved by the finite element method. The deformation for three different load levels φ is plotted in Figure 4. Initially, a stable bending deformation is obtained before compressive overloading of the top face sheet results in development of a wrinkling instability which subsequently turns into a localized instability (Demiray et al. [1]).

## 5. Conclusion

The present contribution is concerned with a higher-order v. Kármán type nonlinear sandwich shell model accounting for transverse flexibility of the core. In a number of analyses of different static and dynamic instability problems, it is found that the transverse core flexibility has distinct effects on the overall structural response of the sandwich panels under investigation

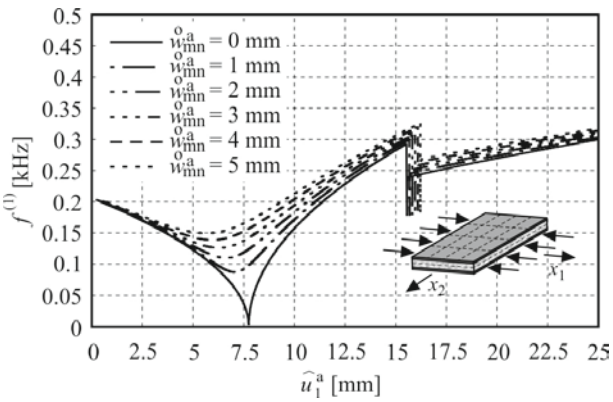


Figure 2: Load and frequency interaction.

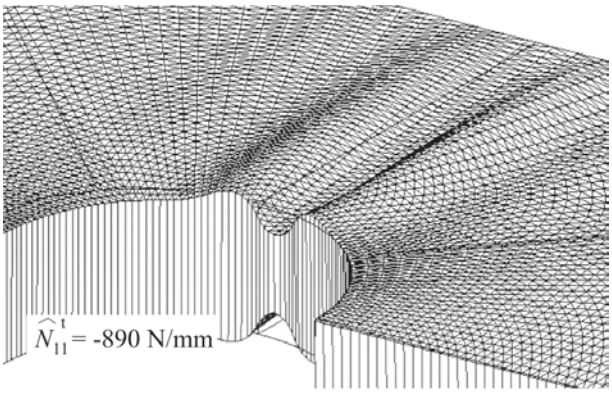


Figure 3: Sandwich plate with central hole under compression..

since it enables the development of local face wrinkling instabilities which may reduce the overall structural stiffness. Discarding these effects in the structural analysis might yield incorrect, in some cases even non-conservative results.

**Acknowledgement**

The present contribution compiles research activities performed over the years by the author together with Liviu Librescu who lost his life in the tragic incident at Virginia Tech on April 16, 2007. This paper is dedicated to his memory.

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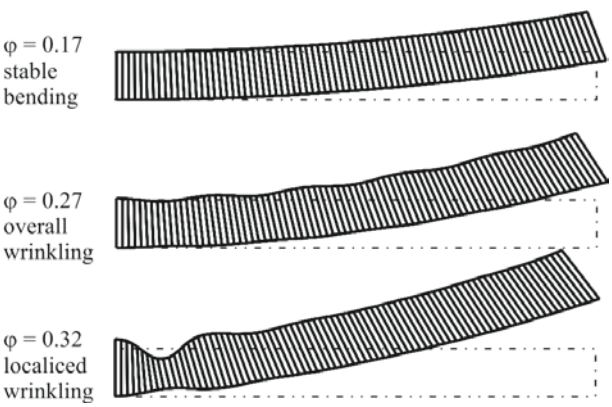


Figure 4: Sandwich beam - local and localized instabilities.