Evolution and verification of a kinematic hypothesis for splitting of the strain energy

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Abstract

Splitting of the strain energy into its "non-membrane" and membrane percentage provides insight into the load-carrying mechanism of structures, subjected to proportional loading. It may be useful, for example, for sensitivity analysis of the initial postbuckling behavior of beams, arches, plates, and shells, and assemblies of such structures. The task of this work is to determine this percentage without computing insignificant numbers such as the values of the strain energy and its membrane part. It is hypothesized that this percentage is proportional to the acceleration of a fictitious particle, moving along a curve on the unit sphere. The curve is described by the vertex of the normalized "fundamental eigenvector" of the so-called "consistently linearized eigenvalue problem". The proportionality factor is obtained from the initial condition for the "non-membrane" percentage of the strain energy, hypothesized as twice the initial velocity of the particle. The lower bound of this factor signals the constancy of this percentage with increasing load, whereas the upper bound indicates a monotonic increase or decrease up to its ab initio predictable value at a stability limit or to an unphysical asymptotic limiting value. The proof of the universal validity of the two hypotheses begins with their verification for the special cases of a membrane stress state and pure bending. The assertion that this is a sufficient condition for the universal validity of these hypotheses is subsequently verified for an example with a monotonically increasing "non-membrane" percentage of the strain energy. It is finally confirmed by an indirect proof of their validity for a non-monotonic, course of this percentage. A by-product of this work are conditions for extreme values of the stiffness of structures, subjected to proportional loading.

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Highlights

- Novel kinematic hypothesis for the "non-membrane" percentage of the strain energy.
- Novel kinematic hypothesis for the initial value of this percentage.
- Kinematic basis of the hypotheses: particle, moving on a curve on the unit sphere.
- Description of the curve by the vertex of a special normalized eigenvector.
- Proportionality of the respective percentage to the acceleration of the particle.
- Verification of the universal validity of the hypothesis.
- Novel conditions for extreme values of the stiffness of structures.

Keywords: "Non-membrane" percentage of the strain energy; Membrane complement of this percentage; Initial value of this percentage; Acceleration vector; Consistently linearized eigenproblem; Extreme values of the structural stiffness

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