Tuning the control system of a nonlinear object with discontinuity by means of the new method of Lyapunov exponents estimation

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Abstract

This work presents the new method of Lyapunov Exponents (LE) estimation and its practical application for control system tuning. By means of the LE estimation method appropriate parameters of the regulator have been found. A mechanical control object - inverted pendulum - has been presented. Equations of the system have been found by Lagrange approach. Identification procedure of the nonlinear control object with discontinuity has been described. From the set of possible parameters of the regulator, the ones of best performance have been selected. The foregoing work proves that proposed method is efficient and results in great performance of the regulator.

Keywords: dynamical system, control, stability, Lyapunov exponent, regulator, tuning, optimization

1. Introduction

Depending on a dynamical system type and a kind of information that is useful for its investigations, different types of indexes or coefficients describing dynamics can be applied. In practise, Lyapunov exponents are one of the most commonly applied tools. These numbers determine the exponential convergence or divergence of trajectories that start infinitesimally close to each other. One of the classical algorithms for calculating Lyapunov Exponents (LE) for continuous systems has been developed by Benettin et al. [2] Alternative method based on synchronization phenomena was elaborated by Stefański [5].

Properties of LE enable their introduction in control systems. The general purpose of a regulator is to force the control object to follow a desired trajectory despite existing disturbances. Therefore, the evaluation and tuning of controllers seems to be a perfect field of LE practical application. Negative value of the Largest Lyapunov Exponent (LLE) ensures stability of the system. Moreover, the lower the value of LLE, the faster average, exponential rate of perturbation rejection.

However, in order to make use of LE in the field of control systems, a fast and simple method of LLE estimation is necessary. Such method has recently been elaborated [3]. Its applicability to control systems has been confirmed by the previous papers [1, 4].

In this paper, the method presented in [3] is applied to improve the performance of a real inverted pendulum by minimizing the LLE value of the closed-loop control system. The control object is nonlinear and discontinuous. The cause of discontinuity is dry friction on bearings. Therefore, applicability of the linear theory is limited. However, it is shown that the method of controller tuning based on LLE can be used successfully for such objects.

2. The method

Assume that an autonomous dynamical system is described by the set of differential equations in the form:

$$\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{f}(\mathbf{x})$$

where **x** is the state vector and **f** is a vector field that (in general) depends on **x**. As it has been shown in the previous work [3], the value of LLE for such system can be estimated from the following expression:

$$\lambda_1^* = \frac{\frac{\mathrm{d}\mathbf{z}_1}{\mathrm{d}t} \cdot \mathbf{z}_1}{|\mathbf{z}_1|^2}$$

where $\mathbf{z}_1$ is a perturbation vector. The approximate value of LLE (λ_1) is obtained by averaging values of λ^*_1 from subsequent computation steps. For long enough time of integration, the average value of λ^*_1 converges to the LLE.

3. The regulation object

The object under consideration is an inverted pendulum presented in Fig. 1. The pendulum bar is attached to a moving cart. The cart is constrained to move in the horizontal direction. The pendulum rod is free to oscillate around a pivot point fixed to the cart. The objective of the system is to control horizontal position of the cart *x(t)* so that the bar remains in the upright vertical position. Such position would be unstable without the appropriate control of the cart position and velocity.

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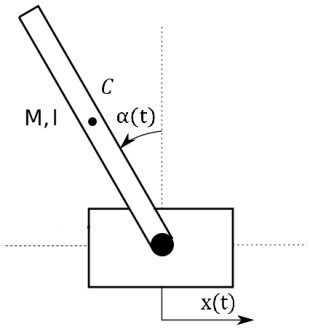

Figure 1: Scheme of the inverted pendulum system

If the drive of the cart is stiff enough, one can assume that the position of the cart *x(t)* is not influenced by the motion of pendulum bar. In such case the description of the regulation object is limited to one equation, which can be easily found using Lagrange approach.

4. Identification of model parameters

In order to perform a realistic simulation of the regulation object, all of its parameters must be estimated with the sufficient accuracy. Firstly, the drive has been identified. For the purposes of identification the velocity of the motor has been recorded after the application of a step input signal. The drive has been identified as a 1st order, linear, time-invariant system. In order to find the values of friction parameters, free vibrations of the pendulum have been investigated. The assumed equation of pendulum free vibrations is as follows:

$$\ddot{\alpha} = -\frac{3g}{2l}\sin(\alpha) - \frac{3b}{Ml^2}\dot{\alpha} - \frac{3c}{Ml^2}\operatorname{sgn}(\dot{\alpha})$$

where b and c are friction coefficients to be identified, g is gravitational acceleration and other symbols are explained in Fig. 1. Such equation does not have an explicit solution. In order to find the values of parameters, nonlinear least-square method has been applied. Fig. 2 presents experimentally recorded free vibrations (dotted line) together with the simulation data (solid line). It can be noticed that the quality of identification is very fine.

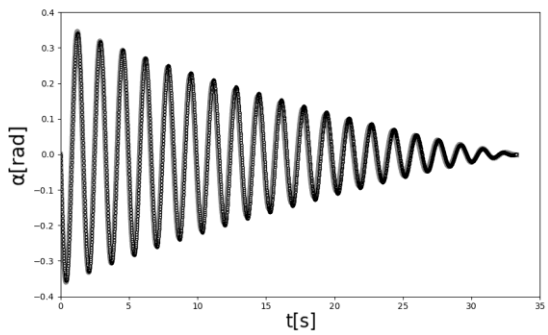

Figure 2: Free vibrations of the pendulum, recorded data (dotted line) and simulation results (solid line)

5. Regulator tuning

The assumed type of regulator is PI (proportional / integral) with additional term proportional to position of the cart:

$$u(t) = kp_1(\alpha + \frac{1}{TI}\int_0^t \alpha d\tau) + kp_2 x$$

where *u(t)* is the output signal and kp_1, kp_2 and *TI* are regulator parameters to be determined. The simulation program has been created to simulate behaviour of the system with an initial perturbation for each set of parameters from a selected range. On each simulation, LLE has been calculated using the method described in paragraph 2. The set of parameters corresponding to the lowest value of LLE ensured that the average, exponential rate of perturbations rejection was as high as possible. The comparison of simulation results between initial guess parameters and the optimized ones is presented in Fig. 3.

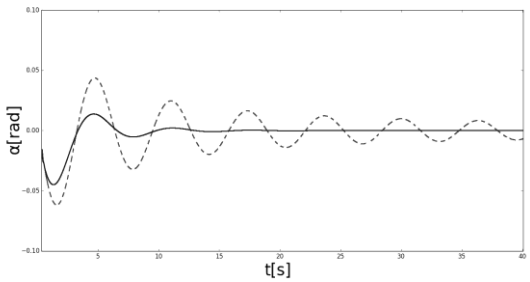

Figure 3: Results of simulation for initial guess parameters (dashed line) and parameters corresponding to the lowest LLE (solid line)

6. Conclusions

It has been confirmed that the new method of LLE estimation can be easily applied for the control system parameters tuning, even if the control object is nonlinear and discontinuous. The lowest value of LLE ensured that the average, exponential rate of perturbations rejection was as high as possible. In future, it is planned to use the method in more sophisticated control systems, such as optimal controllers or neural network controllers.

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