Nonlinear vibrations of a three blades rotor

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Abstract

Dynamics of a rotating structure composed of a rigid hub and three flexible blades is presented in the paper. The nonlinear model of the beam takes into account bending, extension and nonlinear curvature. The impact of geometric nonlinearity on dynamics of the rotating structure is presented in terms of nonlinear modes of natural vibrations and resonance curves of the forced system. The resonance zones are obtained for large oscillations induced by torque expressed by constant and periodic components and supplied to the hub.

Keywords: Nonlinear flexible beam, rotating structure, nonlinear normal modes, vibrations, resonance

1. Introduction

Rotating structures have been studied for many years taking into account different assumptions depending on the beam models. Nonlinear models of thin beams as an extension of classical Bernoulli-Euler theory are presented in [1]. The beam deformations have been obtained assuming the inextensionality condition and a nonlinear curvature which is essential in a case of large oscillations. The model of coupled flexural–torsional vibrations of the rotating beam for helicopter blades applications is proposed in [2], where longitudinal deformations and a nonlinear beam curvature have been taken into account. Recently, a critical review of existing models of the rotating cantilever beams has been published in [3]. Two analytical models, von Karman model and the inextensible beam model have been compared with the finite element method results. The differences between models for selected structural parameters as well as the effect of angular velocity on the softening or hardening phenomenon have been shown.

The development of new composite materials which are nonhomogeneous and also have different mechanical properties in different directions enforced an elaboration of new models of the rotating beams. The models of rotating thinned–walled composite blades with box cross-sections are presented in [4, 5]. Coupled composite beam deformations, nonconstant angular speed and related nonlinear terms are included in the formulation. However, constitutive equations have been assumed to be linear and geometrical nonlinear terms, essential in a case of large deformations, have been neglected.

In this paper a model of a rotor composed of three flexible beams fixed to a rigid hub is studied. The nonlinear beam model is adopted from paper [6] and modified regarding the latest publication [3] and then applied to the full rotor dynamics.

2. Model of the rotating structure

A model of the considered rotating structure is composed of a rigid hub and three flexible blades which may oscillate in the plane of rotation. The beams are modelled by Bernoulli–Euler beam theory. However, due to assumed large oscillations, the geometrical nonlinear terms are taken into account. The experimental setup of the structure is presented in Fig. 1(a). External torque $M(t)$ supplied to the hub includes of constant and periodic components, thus the hub may rotate and oscillate. Depending on amplitude and frequency of excitation as well as structural parameters, the vibration modes of flexible beams may be different than in linear case (Fig. 1b).

In order to derive equations of rotor dynamics we introduce one fixed X_0, Y_0 and rotating x_i, y_i coordinate frames for every single beam. The longitudinal and transversal displacements $u_i(x_i,t)$ and $v_i(x_i,t)$ of an elementary segment dl_i located in point B are defined in the rotating coordinate frame, as presented in Fig. 1(c). Hub rotation is defined by angle $\psi(t)$.

Equations of motion are derived assuming Bernoulli-Euler beam theory and considering that the strain of the elementary segment is defined as: $\epsilon_i = \frac{dL_i - dx_i}{dx_i} = \sqrt{(1+u_i')^2 + v_i'^2} - 1$, where prime is a space derivative with respect to x_i coordinate. Due to large oscillations however, a curvature of the beam is assumed as nonlinear and defined as $\kappa_i = \frac{d\phi_i}{dl_i} = \phi_i' \left(\frac{dl_i}{dx_i}\right)^{-1}$, where ϕ_i represents an angle between axis x_i and a tangent to the deformed beam at point B'. Taking into account the nonlinear curvature we get

$$\phi_i' = \frac{v_i''\left(1+u_i'\right) - v_i' u_i''}{\left(1+u_i'\right)^2 + v_i'^2} \tag{1}$$

Potential energy of a rotor composed of n beams has the form

$$V = \frac{1}{2}\sum_{i=1}^{n}\int_0^{L_i}\left(M_{bi}\frac{\partial\phi_i}{\partial l_i} + N_i\epsilon_i\right)dx_i \tag{2}$$

where $M_{bi} = E_i I_i \frac{d\phi_i}{dl_i}$, $N_i = E_i A_i \epsilon_i$, and $\frac{d\phi_i}{dl_i} = \phi_i'\left(\frac{dl_i}{dx_i}\right)^{-1}$, E_i is Young modulus, I_i a geometrical moment of inertia with respect to the neutral bending axis and, A_i area of a cross-section of the i–beam. Kinetic energy of the whole system is defined as

$$\begin{aligned} T &= \frac{1}{2}J_h\dot{\psi}^2 + \sum_{i=1}^{n}\frac{1}{2}\rho_i\int_0^{L_i}\Big\{\left(\dot{u}_i - \dot{\psi}v_i\right)^2 \\ &+ \left[\left(x_i + R_0 + u_i\right)\dot{\psi} + \dot{v}_i\right]^2\Big\}dx_i \end{aligned} \tag{3}$$

where, J_h is mass moment of inertia and R_0 radius of the hub, ρ_i denotes mass of the i–beam per its unit length.

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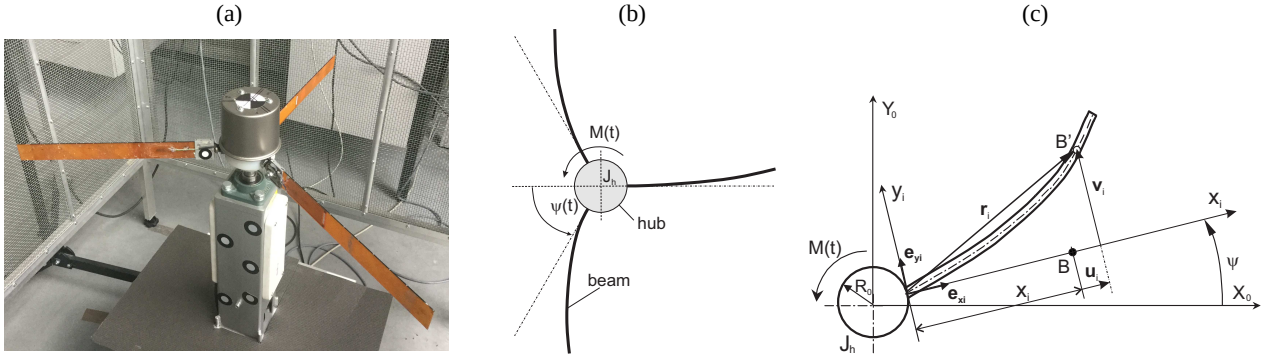

Figure 1: Three blades rotor model: (a) view of the experimental setup, (b) scheme of oscillations of the hub–3 beams structure and, (c) notation and sets of coordinates

The transversal and longitudinal displacements are expressed by small parameter ε assuming that the angle of rotation ψ and transverse displacements v_i are of ε order while longitudinal displacements u_i are of ε^2 order. In the final form for kinetic T and potential V energies we take into account nonlinear terms up to the second order. Substituting them into the extended Hamilton principle of the least action, $\int\limits_{t_1}^{t_2} (\delta T - \delta V + \delta W)\, dt = 0$, where δW is virtual work of external forces, we get a set of partial differential equations of motion (PDEs)

$$\left(\tilde{J}_h + \sum_{i=1}^{n} \tilde{J}_{bi}\right) \ddot{\psi} + \sum_{i=1}^{n} \int\limits_0^{l_i} \varepsilon^2 \Big\{ \left[2\left(R_0 + x_i\right) \dot{u}_i + 2 v_i \dot{v}_i\right] \dot{\psi}$$

$$+ \left[2\left(R_0 + x_i\right) u_i + v_i^2\right] \ddot{\psi} + u_i \ddot{v}_i - v_i \ddot{u}_i + O\left(\varepsilon^3\right) \Big\}\, dx = 0$$

$$\ddot{v}_i + \left(R_0 + x_i\right) \ddot{\psi} + v_i^{''''} + \varepsilon^2 \Big[2\dot{u}_i \dot{\psi} - v_i \dot{\psi}^2 + \frac{5}{2} v_i''^3$$

$$-\alpha_i \left(v_i' \left(u_i' + \frac{1}{2} v_i'^2 \right) \right)' + u_i \ddot{\psi} - 4 v_i'' u_i''' - 6 u_i'' v_i'''$$

$$-10 v_i' v_i'' v_i''' - v_i' u_i^{''''} - 3 u_i' v_i^{''''} - \frac{5}{2} v_i'^2 v_i^{''''} \Big] + O\left(\varepsilon^3\right) = 0$$

$$\varepsilon^2 \Big[\ddot{u}_i - \left(R_0 + x_i\right) \dot{\psi}^2 - 2\dot{v}_i \dot{\psi} - \alpha_i \left(u_i' + \frac{1}{2} v_i'^2 \right)'$$

$$- v_i \ddot{\psi} - v_i' v_i^{''''} \Big] + O\left(\varepsilon^3\right) = 0\,, \quad i = 1 \ldots n \qquad (4)$$

and associated boundary conditions

$$\text{for} \quad x_i = 0, \quad u_i = 0, \quad v_i = 0, \quad v_i' = 0$$

$$\text{for} \quad x_i = l_i, \quad u_i' = 0, \quad v_i'' = 0, \quad v_i''' = 0 \qquad (5)$$

All coordinates and coefficients are dimensionless and normalised to the first beam parameters. Dimensionless time has been defined as $\tilde{t} = \tilde{\omega} t$, where $\tilde{\omega} = \sqrt{\frac{E_1 I_1}{\rho_1 L_1^4}}$ and, $\tilde{J}_h = \frac{J_h}{\rho_1 L_1^2}$, $\tilde{J}_{bi} = \int\limits_0^{l_i} \tilde{\rho}_i \left(R_0 + x\right)^2 dx$, $\tilde{\rho}_i = \rho_i / \rho_1$, $l_i = L_i / L_1$, $\alpha_i = \frac{A_i L_i^2}{I_i}$.

The first of Eqs. (4) represents dynamics of the hub coupled by nonlinear and inertia terms with dynamics of the beams which are governed by the second and the third of Eqs. (4). A number of equations depends on number of blades, in a case of three blades rotor we get a set of seven coupled nonlinear PDEs.

3. Concluding remarks

The nonlinear model of the rotating hub–beams structure has been derived considering extended Bernoulli-Euler beam theory by taking into account nonlinear geometrical terms arising from large oscillations. The model is given by a set of coupled nonlinear PDEs which include transverse and longitudinal oscillations of the beams as well as hub dynamics. The hub plays an essential role because it couples motion of all blades. The challenging task is to find an analytical solution of the mathematical model. It can be done by a direct attack of PDEs or to elaborate the method of the problem reduction from PDEs to ODEs. The latter approach may be based on a linear or nonlinear modes projection with further simplification neglecting longitudinal inertia terms, which can be accepted only for thin beams. In the final formulation rotating beam dynamics will be reduced to transverse oscillations presuming a proper inclusion of longitudinal loading. Then the obtained nonlinear ODEs of the full hub–blades structure will enable analytical and numerical investigations of resonance zones and bifurcation scenarios. Both mentioned approaches will be applied in order to asses solutions by different approaches.

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