Three-dimensional analysis of sandwich plates with functionally graded cores using a two-dimensional numerical model

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Abstract

This work presents a numerical analysis of a sandwich plate (SP) with functionally graded (FG) core. The value of Young's modulus changes along the thickness of the SP core. A full three-dimensional (3D) model is applied to the numerical model which only uses a two-dimensional (2D) in-plane mesh. The method is called FEM23, which means the 2D finite element method for a 3D model. A special post-processing procedure, tailored specifically for FEM23, has been developed, which enables visualisation of all 3D results. In FEM23, the spatial approximation is split into two – in-plane and transverse – parts. The order of transverse approximation for each layer of the SP is appropriately adjusted. For the thick FG core, the 4th order approximation along the thickness is applied whereas for the SP faces, the 1st order is deemed sufficient. The method is illustrated with a set of examples. In some of the examples, the results are compared with other numerical or analytical solutions. The examples confirm the correctness, robustness and flexibility of FEM23 for SP.

Keywords: sandwich plate, functionally graded material, FEM23

1. Introduction

In recent years, functionally graded (FG) materials have found use in a wide range of modern applications. These kinds of materials are used in structures which must satisfy very strict requirements, for example due to severe operating conditions. The mechanical properties of FG materials change smoothly along certain direction. It allows them to maintain high specific stiffness in contrast to classical composites where there are stress discontinuities at layer-interfaces. A great number of theories have been proposed by researchers in order to properly describe the mechanical behaviour of FG materials, see [1].

The sandwich plate (SP) analysed in this paper consists of two faces (upper and lower thin layers) and the middle thick layer, here referred to as the core. The core is made of a FG material, see Fig. 1. Poisson's radio is constant in the core, whereas Young's modulus changes along the plate thickness. A two- dimensional (2D) numerical model has been formulated for a robust full three-dimensional (3D) numerical analysis using FEM23 (3D finite elements method on a 2D mesh) [2, 3]. It means that although only a 2D discretisation is performed for the whole SP, yet full 3D results are obtained.

FEM23 is particularly suitable for use in analysis of homogeneous plates. In this method, the in-plane approximation order results from the orders of finite elements (FE) in a 2D mesh, while in the transverse direction the approximation may change from the 1st up to the 4th order, depending on the plate thickness. Afterwards, if a smart assembling procedure in FEM23 is used, the analysis may be employed for layered plates as well. FEM23 is completed with special post-processing in which full 3D results may be visualised.

In the analysed SP, the skins are very thin, whereas the core is thick in relation to other dimensions of the plate. Thus, in FEM23 the 1st order of transverse approximation for skins is sufficient. For the core, the 4th order approximation along its thickness has to be used to accommodate the non-uniform material properties. In the final results, stress jumps are expected in the face-core interfaces.

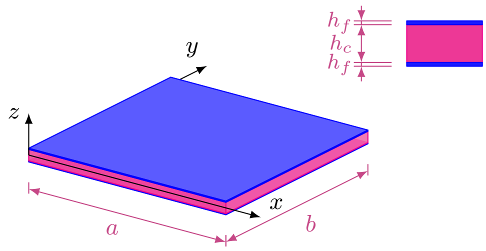

Figure 1: Sandwich plate.

2. Mathematical model

As has been mentioned in the Introduction, FEM23 for multi-layered structures initially dedicated for homogeneous materials now can also be used for functionally graded (FG) materials along plate thickness. The mathematical model of the considered problem is composed of a variational equilibrium equation with appropriate boundary conditions:

$$\begin{aligned} &\int_{S^\sigma} \mathbf{v}\cdot\hat{\mathbf{t}}\,\mathrm{d}S - \int_V \boldsymbol{\varepsilon}(\mathbf{v}) : \mathbf{E} : \boldsymbol{\varepsilon}(\mathbf{u})\,\mathrm{d}V = 0 \\ &\mathbf{u} = \hat{\mathbf{u}} \quad \text{on } S^D \end{aligned} \tag{1}$$

where $\mathbf{v}$ is the test function with condition $\mathbf{v} = \mathbf{0}$ on S^D, $\boldsymbol{\sigma}$ is the stress tensor, $\hat{\mathbf{t}}$ is the prescribed traction forces vector, $\mathbf{u}$ is the displacement vector and $\hat{\mathbf{u}}$ is the prescribed displacement vector, S^D is the part of the boundary where displacement $\hat{\mathbf{u}}$ is prescribed, $\mathbf{E}$ is Hooke's tensor, which for SP core changes along thickness. The Cauchy strain is used in eq. (1) that is defined as symmetric part of the gradient tensor $\boldsymbol{\varepsilon}(\mathbf{v}) = \boldsymbol{\nabla}_s \mathbf{v}$.

A clear distinction needs to be made in FEM23 between in-plane and transverse directions. It refers to spatial approximation integration and differentiation. In this context, eq. (1) may be rewritten to adopt the following form:

$$\begin{aligned}
&\int\limits_{S^m} \int\limits_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\partial \mathbf{v}}{\partial z} \cdot \mathbf{E}_1 \cdot \frac{\partial \mathbf{u}}{\partial z} \, \mathrm{d}z \, \mathrm{d}S \\
&+ \int\limits_{S^m} \int\limits_{-\frac{h}{2}}^{\frac{h}{2}} \left(\frac{\partial \mathbf{v}}{\partial z} \cdot \mathbf{E}_2 : \bar{\boldsymbol{\nabla}} \mathbf{u} + \bar{\boldsymbol{\nabla}} \mathbf{v}^{\mathrm{T}} : \mathbf{E}_3 \cdot \frac{\partial \mathbf{u}}{\partial z} \right) \mathrm{d}z \, \mathrm{d}S \qquad (2) \\
&+ \int\limits_{S^m} \int\limits_{-\frac{h}{2}}^{\frac{h}{2}} \bar{\boldsymbol{\nabla}} \mathbf{v}^{\mathrm{T}} : \mathbf{E}_4 : \bar{\boldsymbol{\nabla}} \mathbf{u} \, \mathrm{d}z \, \mathrm{d}S = \int\limits_{S^\sigma} \mathbf{v} \cdot \hat{\mathbf{t}} \, \mathrm{d}S
\end{aligned}$$

where S^σ is the part of the boundary where tractions $\hat{\mathbf{t}}$ are prescribed, $\mathbf{E}_i$ are the appropriate parts of the Hooke's tensor, $\bar{\boldsymbol{\nabla}}$ is the in-plane gradient operator, h is the thickness of plate and S^m is the in-plane surface.

Every integral along the plate thickness is substituted by closed Gauss integration schemes

$$\int\limits_{-\frac{h}{2}}^{\frac{h}{2}} g(x, y, z) \, \mathrm{d}z = \sum_{i}^{n_g} w_i \, g(x, y, z_i) \qquad (3)$$

where n_g is the number of integration points, z_i are the integration points and w_i are the integration weights. The order of Gaussian quadrature in eq. (3) has to be adjusted to the order of transverse approximation and level of material grading. Taking into account eq. (3) the eq. (2) is described only on S^m surface in which the 2D FE mesh is constructed.

3. Approximation

FEM23 requires special 3D spatial approximation in which the in-plane and transverse components are strictly decomposed. The first up to fourth order transverse approximation may be used, but – for the sake of clarity – only the 1st order is presented here. In that case the displacement 3D approximation takes the following form:

$$\mathbf{u}(x, y, z) = N_1(z) \mathbf{u}^u(x, y) + N_2(z) \mathbf{u}^l(x, y) \qquad (4)$$

where $\mathbf{u}^u$ and $\mathbf{u}^l$ are the displacement vectors on upper S^u and lower S^l plate surfaces, respectively. Functions N_1 and N_2 are 1D first order Lagrange polynomials. The displacements vectors $\mathbf{u}^u$ and $\mathbf{u}^l$ are approximated in space (x, y) by a shape function in 2D mesh, and these vectors are approximated by the same shape functions $\boldsymbol{\Phi}(x, y)$ constructed in the 2D mesh:

$$\mathbf{u}^u(x, y) = \boldsymbol{\Phi}(x, y) \check{\mathbf{u}}^u \, , \qquad \mathbf{u}^l(x, y) = \boldsymbol{\Phi}(x, y) \check{\mathbf{u}}^l \qquad (5)$$

where $\check{\mathbf{u}}^u$ and $\check{\mathbf{u}}^l$ are vectors of degrees of freedom for vectors $\mathbf{u}^u$ and $\mathbf{u}^l$, respectively.

4. Examples

A series of examples have been presented in this work. Some of them have been taken from other sources, e.g. from [4, 5]. In one of the examples, following [4], the subject of analysis is a simply supported beam. Two cases of Young's moduli change in the core are assumed, as shown in Fig. 2. This example is calculated as SP with planar dimensions $a \times b = 0.3 \times 0.06$ m. The thickness for the core and faces are $h_c = 20$ mm and $h_f = 0.3$ mm, respectively. Young's moduli for the material used for faces is $E_f = 50$ GPa and Poisson's radio for the faces and core are $\nu_f = 0.25$ and $\nu_c = 0.35$, respectively. The plate is subjected to uniform pressure – case (a) or sinusoidal pressure – case (b) . The result of the analysis by FEM23 in the form of map of σ_{xx} on the deformed plate for case (b) is presented in Fig. 3. The stresses are concentrated on the faces, whereas the level of stresses in the core are hundreds times smaller. The obtained results are compliant with those shown in [4].

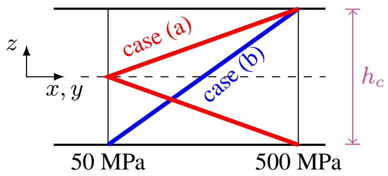

Figure 2: Variation of E_c for sandwich plate.

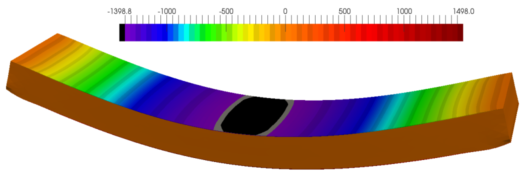

Figure 3: Map of σ_{xx} [Pa] on deformed sandwich plate, case (b).

5. Conclusions

In this work, bending of an SP with a FG core is numerically analysed by FEM23. The faces of the SP are very thin and, on the other hand, the core is thick and FG along its thickness. A full 3D analysis is obtained, although only a 2D in-plane mesh is used. The accuracy of the method has been tested with the use of some known solutions as well as by comparison with the analytical solution and the well-established FEM procedures.

References

[1] Swaminathan, K., Naveenkumar, D.T., Zenkour, A.M. and Carrera, E. Stress, vibration and buckling analyses of FGM plates–A state-of-the-art review *Compos. Struct.*, 120, pp. 10–31, 2015.

[2] Jaśkowiec, J., Pluciński, P. Three-dimensional modelling of laminated glass bending on two-dimensional in-plane mesh *Compos. Struct.*, 171, pp. 562-575, 2017.

[3] Jaśkowiec, J., Pluciński, P., Stankiewicz, A., Cichoń, C. Three-dimensional modelling of laminated glass bending on two-dimensional in-plane mesh *Composites Part B*, 120, pp. 63-82, 2017.

[4] Filippia, M., Carrera, E., Zenkourb, A.M. Static analyses of FGM beams by various theories and finite elements *Composites Part B*, 72, pp. 1–9, 2015.

[5] Nguyen, T-K at al. Static and vibration analysis of isotropic and functionally graded sandwich plates using an edge-based MITC3 finite elements *Composites Part B*, 107, pp. 162–173, 2016.