ACTIVE CONTROL OF HALF CAR SUSPENSION SYSTEM BASED ON LINEAR QUADRATIC REGULATOR

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Abstract

In this paper analysis of a half-car model with hybrid dampers is performed. Differential equations in a matrix form were written. The value of the active control force, its power and its limits was determined. Using Matlab-Simulink software, the parameters of the optimal hybrid system were determined. Used in consider LQR controller, which can be considered one of the solutions for excellent comfort ride and good handling of cars.

Keywords: hybrid damper, vibration damping, LQR controller, active reduction vibrations

1. Introduction

The main purpose of the damper is reduce the vibrations. Elimination of this vibrations is very important for the safety of the driver and passengers. When driving a bumpy road the car can be detached from the surface of the road so it is important that the shock absorbers do their job well. The hybrid damper is a combination of active and passive part.

2. Analytical half-car suspension model

A half-car of the mass m and moment of inertia I is considered, which moves at a velocity *v* on the unevenness of the road. The function $w_1(t)$ and $w_2(t)$ define the applied kinematic excitation describing the unevenness of the road surface. The relation between $w_1(t)$ and $w_2(t)$is the following equation:

$$w_2(t) = w_1(t - t_0) \tag{2.1}$$

Distance between both vehicle axles is following $d = d_1 + d_2$, where d_1 - is distance between front vehicle axle and *y* position of the mass centre of the car, d_2 - is distance between rear vehicle axle and position of the mass center of the car. The delay time t_0 is related to driving velocity *v* and distance *d*. The parameters c_1, c_2, c_3, c_4 define damping coefficients of the considered model. The parameters k_1, k_2, k_3, k_4 define stiffness coefficients of this model. Additionally, it is assumed of small displacement.

The half-car suspension model is shown in Figure 1. Equation describing vibration [4] can be written in a matrix form:

$$M\ddot{y} + C\dot{y} + Ky = \tilde{B}u + \tilde{G}w(t) \tag{2.2}$$

where:
M - the mass matrix,
C - the damping matrix,
K - the stiffness matrix,

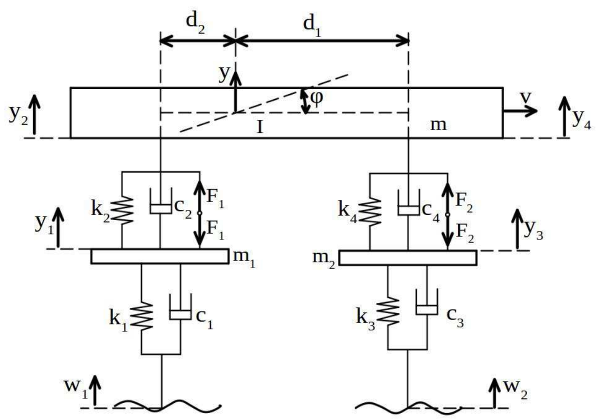

Figure 1. Half-car suspension model.

3. Hybrid dampers.

These equations 2.2 can be simply written as a matrix equation:

$$\dot{x} = Ax + Bu + Gw \tag{3.1}$$

where the state vector x is composed of:

$$x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

$$y = [y_1, y_2, y_3, y_4]^T$$

$$\dot{y} = [\dot{y}_1, \dot{y}_2, \dot{y}_3, \dot{y}_4]^T$$

The input vector u representing the two actuator forces, while the disturbance vector w consists of road disturbance.

$$u = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$w = [w_1, w_2, \dot{w}_1, \dot{w}_2]^T$$

The matrix representation of Equation 2.2 would be the basis for linear optimal controller design. The linear time-invariant

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system, (LTI), is described by Equation 3.1. For controller design it is assumed for the purpose of this summary only, that all the states are available and also can be measured exactly. First of all let us consider a state variable feedback regulator:

$$u = -Kx \quad (3.2)$$

where K is the state feedback gain matrix. The optimization procedure consists of determining the control input u, which minimizes the performance index. The performance index J represents the performance characteristic requirement as well as the controller input limitations. In this paper approaches are taken in order to evaluate the performance index, and hence designing the optimal controller shown in [5,6]. In this method, the performance index J penalizes the state variables and the inputs; thus, it has the standard form of:

$$J = \int_0^\infty (x^T Q x + u^T R u) dt \quad (3.3)$$

where Q and R are positive definite, being called weighting matrices. Here the passenger acceleration which is an indicator of ride comfort is not being penalized. Linear optimal control theory provides the solution of Equation 3.3 in the form of Equation 3.2.

The gain matrix K is computed from:

$$K = R^{-1} B^T P$$

where the matrix P is evaluated being the solution of the Algebraic Riccati Equation, (ARE).

$$AP + A^T P - PBR^{-1} B^T P + Q = 0$$

4. Numerical results.

Below there is a graph showing the results of the numerical calculations for the half-car model. The acceleration waveforms for the model without LQR optimization were presented.

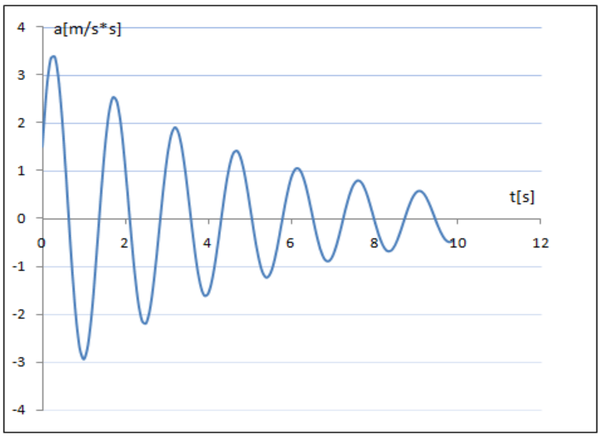

Figure 2. Acceleration of a mass for the half-car model with no control force.

5. Conclusions.

The paper shows that an active suspension gives a better performance in terms of comfort ride compared to the passive suspension. An active suspension also increases a tire to road contact in order to make the vehicle more stable. This concludes that LQR controller can be considered one of the solutions for excellent comfort ride and good handling of cars. According to [5,6], the LQR controller gives a better performance in terms small percentage overshoot and faster settling time comparing to the passive system. Generally, an active suspension has an actuator connected between the sprung and unsprung masses of the vehicle.

Illustration of the active suspension system is shown in (Figure 1). The system consists of actuator mounted between car body and car wheel. The actuator can generate control forces which are calculated by a computer to suppress the system responds to the changes of the road condition. Active suspension system has an ability to store, dissipate and to introduce energy to the system. The actuator is connected in parallel with a spring and absorber, while the sensors of the body are located at different points of the vehicle to measure the motions of the body.

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