Analytical and experimental modal analysis of a composite circumferentially asymmetric stiffness box beam

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Abstract

This paper presents a theoretical vs. experimental modal analysis of a composite thin-walled beam featuring a circumferentially asymmetric stiffness characteristics. The adopted lamination scheme results in the complex elastic deformation modes exhibiting coupling of flapwise bending, transverse shear and torsion. The analytical model used in this study is based on the previous author research. The theoretical results are compared to experimental ones obtained by two different methods of analysis, namely an impact hammer test and a laser vibrometer test. The performed tests show that the use of a laser-based motion analysis system is a feasible and accurate method for static and dynamic experiments on flexible systems, especially the ones that can not be measured by conventional contact methods. Therefore, this measuring technique will be used in future modal analysis experiments on a rotating beam.

Keywords: modal analysis, circumferentially asymmetric stiffness beam, bending-twisting coupling, laser vibrometer

1. Introduction

Thin-walled beams made of composite materials are structural elements of many modern constructions of aeronautical/aerospace, automotive, civil and naval engineering. Typical examples are robotic arms, lightweight satellite antena structure appendages, aircraft wings, helicopter and turbomachinery rotor blades etc. The primary interest in structural analysis is usually aimed at cantilever specimens bending effect, often coupled to torsional deformation and/or lateral bending. Moreover, specific cases of tapered, twisted, pre-stretched or rotating elements are investigated too. Typically the research is focused on blades static and dynamics characteristics; the later one involving the natural vibration frequencies and deformation mode shapes.

Over the recent two decades many advanced analytical models have been developed to analyse theoretically the performance of composite beams with a solid as well as a thin-walled cross-section. References by Giavotto et al [4], Crespo da Silva [2], Hodges [6] and Librescu and Song [9] are some representative examples that are aimed at a detailed structural analysis of beams featuring coupled deformation modes. A comprehensive discussion that examines the various assumptions in these models and the quality of their results may be found in review papers e.g. by Hodges [5] and Volovoi et al. [10].

An essential part of any comprehensive structural analysis is a laboratory experiment that is an ultimate test to asses the accuracy of the derived analytical model. Experimental results give rise to formulation refinements in terms of various model parameters and phenomena that are taken into account. Some experiments on composite cantilever beam vibration analysis and the computation of its natural frequencies and mode shapes are presented in e.g. [1], [8].

In the present research a free-vibration behaviour of a composite box cross-section beam with the elastic deformation couplings is examined. Based on the previous author research presented in [3], a set of governing partial differential equations for combined flapwise bending, transverse shear and torsional motion of the blade is written and next reduced to the ordinary differential equation by adopting the derived orthogonality condition [7]. Finally, this equation is solved for natural frequencies and deformations mode shapes. These analytical results are compared to experimental one as obtained by testing the box-cross section specimen in the laboratory.

2. Problem formulation

Let us consider a slender, straight and elastic composite thin-walled box beam clamped at the rigid wall. The length of the blade is denoted by l, its cross-section width and depth are d and c, respectively. The specimen wall thickness is denoted by h and it is assumed to be constant spanwise. The discussed blade is made of a linearly elastic multilayered laminate with reinforcing fibres placed according to the circumferentially asymmetric stiffness (CAS) scheme [9]. This configuration results in bending-transverse shear-twisting deformation coupling the magnitude of which depends on the reinforcing fibres orientation.

2.1. Governing equation

An analytical model of the structure under consideration is based on the theory developed by Librescu and Song [9]. The full system of partial differential equations governing the dynamics of the discussed beam for a general case of a slewing motion about vertical axis is given in papers [3, 7]. This system is transformed into ordinary differential equations form taking into account the normal modes projection and an associated orthogonality condition. To this aim the Galerkin procedure for the appropriate natural mode is applied. Next, the system is converted to the dimensionless notation and the final form is as follows

$$\ddot{q} + \alpha_2\ddot{\psi} + (\alpha_1 + \alpha_3\dot{\psi}^2)q + \alpha_4 q\dot{q}\dot{\psi} = 0 \tag{1}$$

where q is the generalized coordinate corresponding to the studied coupled flexural–torsional mode, coefficients α_j $(j = 1, 2, 3, 4)$ are obtained from the modes projection and $\psi(t)$ is the temporary beam position angle resulting from its rotation about vertical axis crossing clamped end.

3. Results

To verify the discussed analytical model of the thin-walled beam a test specimen with bending-twist coupling was fabricated out of graphite-epoxy unidirectional prepreg using an autoclave molding technique. The laminate stacking sequence of the blade is $[90_2/15_4/90/15_4/90_2]_{\mathrm{T}}$ where the fibre orientation angle is measured from the spanwise axis. According to the study presented in [7] the orientation of reinforcing fibres at 15° angle results with maximum bending-twist elastic deformation coupling. However, several additional layers set at 90° were required to provide the necessary stiffness of the material at the mid line of profile webs.

The cross-section dimensions of the beam are $80 \times 20\,\mathrm{mm}$, the length l is $900\,\mathrm{mm}$ and the wall thickness $h = 0.8\,\mathrm{mm}$. The physical properties of the unidirectional graphite-epoxy prepreg material are as follows: Young modulus along the fibres $E_1 = 170.6\,\mathrm{GPa}$, transverse Young modulus $E_2 = 7.6\,\mathrm{GPa}$; Poisson ratio $\nu_{12} = 0.36\,[-]$; shear modulus $G_{23} = 5.05\,\mathrm{GPa}$; shear modulus $G_{12} = G_{13} = 3.5\,\mathrm{GPa}$; mass density $\rho = 1324\,{}^{\mathrm{kg}}/_{\mathrm{m}^3}$.

3.1. Analytical calculations

Numerical values of first three natural frequencies as calculated from eqn. (1) are put in Table 1, col. 2. As reported in the previous section, these are complex deformation modes with different relative importance of individual components dependent on mode order. Figure 1 presents a detailed plot of modes components for two lowest natural frequencies (32.89 Hz and 189.94 Hz).

Table 1: Frequencies of natural vibrations of the CAS beam (Hz)

	Analytical	Experiment	
		Impact test	Laser vibrometer
Mode 1	32.89	31.41	31.87
Mode 2	189.04	183.77	184.37
Mode 3	389.62	429.66	437.51

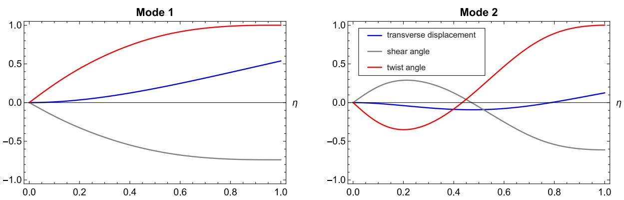

Figure 1: Components of beam deformations corresponding to first two natural modes

3.2. Experimental setup and results

The given above analytical outcomes were compared to experimental results obtained by using two different methods of analysis, namely an impact hammer test and a laser vibrometer measurement.

For the hammer test the blade was fixed through a dedicated mount bracket to the anti-vibrational TIRA TGT MO 48 XL monobase slip table. The impact tests were performed with a PCB Piezotronics modal hammer model 086E80 and a PCB Piezotronics accelerometer 352M208. Both devices were connected to the LMS SCADAS III modal analyser with PQA II data acquisition card; for data processing a TestLab 10B software was used. 54 measuring points were marked on the tested structure and the acceleration sensor was positioned close to the beam end. Each point was excited several times so the final results were obtained based on the results of approx. 250 excitations. Findings are summarised in Table 1, col. 3; Figure 2 shows the approximation of the first natural mode as received from the LMS system.

Preparing for the laser scanning experiment a series of reflective markers was adhered to the beam outside surface. During the test the instantaneous 3-D coordinates of these points were traced by the laser scanning head to capture the dynamically deformed geometry of the specimen. This could be done based on the Doppler effect relating the measured frequencies of an emitted and reflected back laser beam as well as specimen velocities with respect to the beam camera. The received signal was converted to the frequency domain by using a fast Fourier transform (FFT) and natural frequencies and modes of the tested structure were obtained. In the laser vibrometer experiment the PSV-500 camera system by Polytec was used. Results are gathered in the last column of Table 1.

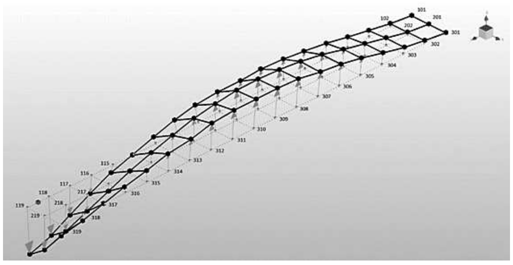

Figure 2: Plot of the fundamental mode as received from the LMS data acquisition system

4. Final remarks

This study discussed the analytical and experimental modal analysis of a thin-walled circumferentially asymmetric stiffness box beam featuring the complex bending-torsional coupled deformation. Natural frequencies and shape modes were extracted. Analytical results and both experiments findings show a satisfactory correlation. Moreover, the laser-based motion analysis system was confirmed to be a feasible and accurate method for dynamic experiments on flexible systems. This is an important conclusion with respect to the planed further tests on rotating structure where the conventional contact methods of modal analysis can not be applied.

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