# Shear banding in large strain plasticity – influence of specimen dimensions

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Abstract

The research presented in this paper is focused on localization simulations for elastoplastic material undergoing large strain in isothermal conditions in a plate under tension assuming plane strain or plane stress state. The attention is concentrated on the dimension and mesh dependence of the numerical results, especially on finding correlation between thickness of the specimen and the width of shear bands. First different lengths of the specimen with fixed cross-section and three meshes are considered. For the plane stress case different thicknesses and different numbers of elements in the thickness direction are also studied. Numerical simulations are performed using AceGen and AceFEM packages.

*Keywords: plasticity, large strains, strain localization*

## 1. Introduction

The elongation process for an elastoplastic material with large deformation often leads to the loss of stability due to geometrical effects. Strains might then localize in a certain zone of elements in the specimen forming a neck or a shear band. Simulations are performed for a plate in tension and strain localization is induced by geometrical softening caused by large strains. The following assumptions are made in the analysis: static loading, hyperelasticity, rate-independent plasticity with associated flow rule and no hardening. In the paper two stress states are considered: plane strain and plane stress. Three-dimensional formulation is employed. The influence of different dimensions of the specimen on the results is examined with special attention paid to the width of localization band and its relation with the thickness of the specimen. Similar study was performed in [2] where the analysis of strain localization for a tensile bar with rectangular cross-section and large strain with plastic hardening was considered. The author applied gradient-enhanced 2D plane stress model with incorporated length scale which depends on the specimen thickness and compared the results with a full 3D model.

## 2. Finite elastoplasticity model

In this work a multiplicative decomposition of the deformation gradient into its elastic and plastic parts is considered [4]

$$\mathbf{F} = \mathbf{F}^e\mathbf{F}^p \tag{1}$$

The free energy potential is assumed in an additive form

$$\psi(\mathbf{b}^e, \gamma) = \psi^e(\mathbf{b}^e) + \psi^p(\gamma) \tag{2}$$

where the first part represents elasticity and the second part plastic hardening. $\gamma$ is a plastic strain measure. The formula for the elastic left Cauchy-Green tensor is presented below

$$\mathbf{b}^e = \mathbf{F}^e(\mathbf{F}^e)^{\mathrm{T}} \tag{3}$$

The yield condition is formulated in a classical way

$$F_p(\boldsymbol{\tau}, \gamma) = f(\boldsymbol{\tau}) - \sqrt{2/3}\sigma_y(\gamma) \leq 0 \tag{4}$$

where $f(\boldsymbol{\tau})$ is an equivalent Kirchhoff stress function and $\sigma_y(\gamma)$ denotes the yield strength. The Huber-Mises-Hencky (HMH) definition is used in this paper

$$f = ||\boldsymbol{\tau}^{dev}|| \tag{5}$$

The governing equation (the balance of linear momentum), written in Euler description, can be found in [4]. The numerical simulations are carried out using the symbolic-numerical packages *AceGen* and *AceFEM* [1] for *Wolfram Mathematica*. The important advantage of the former package is automatic differentiation which enables an easy derivation of the tangent operator.

## 3. Results

In this paper shear banding in a plate in tension is examined. Only one eighth of the plate is considered due to symmetry. The deformation in the width direction is free and in the length direction extension with the maximum value of 0.08 m, scaled by factor $\lambda$, is imposed. To initiate localization an imperfect element is introduced in the plate centre. The displacements in the thickness direction are constrained for the plane strain case and free for plane stress.

Selected results for the plane stress case simulated with ideal plasticity are presented in Figures 1-5. The plane strain case is considered in [5]. The simulations are performed for three meshes: coarse (mesh1), medium (mesh2) and fine (mesh3). The basic plate dimensions are: length L=0.4 m, width W=0.2 m, thickness H=0.01 m (dimension proportions 40×20×1). The analysed proportions of the specimen are as follows: 20×20×1, 40×20×1, 80×20×1, 40×20×2, 40×20×4.

In Figure 1 the diagrams relating the sum of reactions to the extension multiplier are presented for different lengths with fixed cross-section and the medium mesh. The diagrams differs slightly in the elastic range and close to the peak, then the localized deformation does not depend on specimen length. In Figure 2 the deformed meshes (mesh1) at the end of the elongation process are presented. Similar shear bands are obtained for three different specimen lengths.

The results for different meshes are shown in Figure 3. Strong mesh sensitivity and more ductile response for coarser meshes

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can be observed.

Simulations for different thicknesses are shown in Figure 4. For a thicker specimen rather necking than a shear band is obtained. Small mesh sensitivity of the diagrams is noticed. Figure 5 presents the unexpected deformation pattern obtained for the plate 4 cm thick. Two shear bands are observed also for the medium mesh and for smaller loading steps.

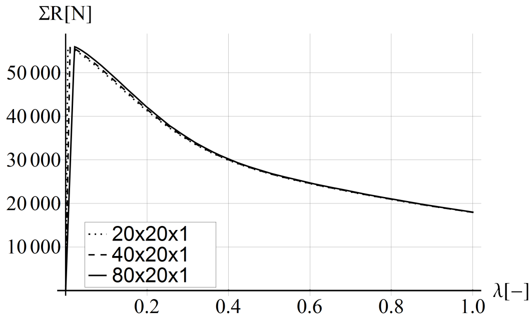


Figure 1: Sum of reactions vs displacement factor for plane stress, ideal plasticity case and different dimensions (medium mesh)

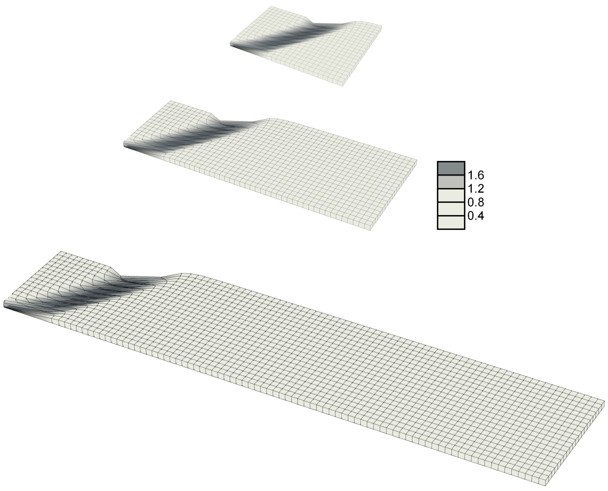


Figure 2: Deformed coarse mesh at the end of elongation process for plane stress, ideal plasticity case and plate proportions 20×20×1 (top), 40×20×1 (middle), 80×20×1 (bottom)

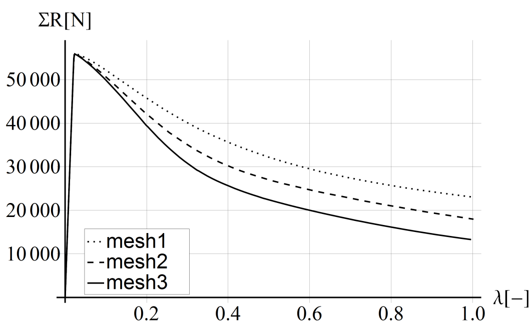


Figure 3: Sum of reactions vs displacement factor for plane stress, ideal plasticity case and 80×20×1

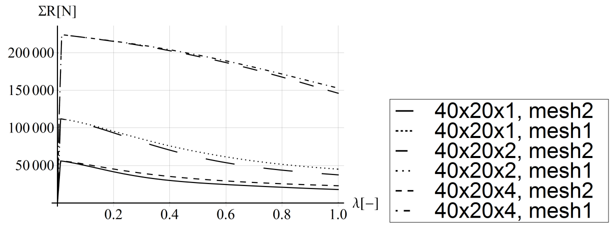


Figure 4: Sum of reactions vs displacement factor for plane stress, ideal plasticity case and different plate thicknesses

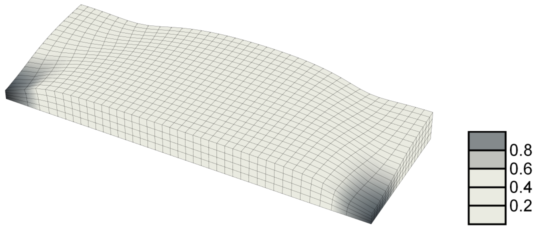


Figure 5: Deformed coarse mesh at the end of elongation process for plate proportions 40×20×4

## 4. Conclusions

This paper presents the analysis of the large strain, elastoplastic model for a plate under tension. The geometrical instabilities result in localized deformation modes and the considered boundary value problem can lose well-posedness. Thereby spurious mesh sensitivity of simulation results can be observed. Hence a localization limiter for the isothermal model is necessary. It can be provided by a gradient enhancement of the constitutive description [3].

On the other hand, the results obtained so far have not shown a clear dependence of the width of the shear bands on the thickness of the analysed plate. Further research is necessary.

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