

Optimal positions and parameters of translational and rotational mass dampers in beams subjected to random excitations

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Abstract

The paper discusses the problem of random vibration of a beam with mass dampers attached. The Euler-Bernoulli beam is subjected to concentrated and distributed forces with peaked Power Spectral Densities. The equation of motion is solved using the Fourier method and the time Laplace transformation. The power spectral characteristics of the beam deflection are determined. The aim of the paper is to study the problem of optimal positions and parameters of the translational and rotational mass dampers due to the vibration suppression. The numerical calculations for the Root Mean Square of the beam deflection taken as the objective function are performed.

Keywords: dynamic vibration absorber, beam vibration suppression, random excitations, Power Spectral Densities

1. Introduction

Mass dampers (of passive, semi-active or active type) are widely used to suppress the motion of a structure mainly at the point of their attachment [4]. In civil engineering they are used to control vibration induced by wind flow, ground motion or traffic in bridges, high-rise buildings, masts, wind turbine towers, and footbridges [1,6].

In continuous structures, in large part of the practical cases, the best locations of dampers are the points of loading application, and they should not be attached in nodes of the vibration mode with a natural frequency to which the dampers are tuned [2].

These rules apply to translational dampers. However, in structures that transfer bending moment, like beams, a rotational mass damper can be used, significantly improving vibration damping efficiency [3,5]. The rotational damper can be effective even when located in a node.

In this article the model of a beam subjected to concentrated and distributed random excitations with Power Spectral Densities (PSD) is presented. A system of translational and rotational mass dampers is attached to the beam. The local optimization problem (minimization of displacement at a given point) and global optimization problem (minimization of the whole beam energy) are considered.

2. Theoretical model

Figure 1 presents a beam subjected to concentrated and distributed forces with a system of translational and rotational mass dampers attached. Assuming the Euler-Bernoulli theory and the Voigt-Kelvin rheological model the equation of motion is given by

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \alpha \frac{\partial^3 w}{\partial x^4 \partial t} + EI \frac{\partial^4 w}{\partial x^4} = q(x, t) + \sum_{j=1}^p P_j(t) \delta(x - x_j^O) + \sum_{j=1}^r F_j(t) \delta(x - x_j^E) + \sum_{j=1}^r \frac{\partial M_j(t) \delta(x - x_j^E)}{\partial x} \quad (1)$$

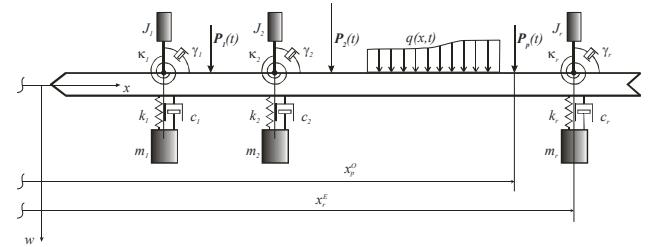


Figure 1: Beam with system of translational and rotational mass dampers

Solution to Eqn (1) is assumed in the form of a series

$$w(x, t) = \sum_{i=1}^{\infty} q_i(t) \varphi_i(x) \quad (2)$$

Functions $\varphi_i(x)$ in Eqn (2) are the modes of vibration of the beam without dampers. Performing the time Laplace transformation results in

$$\sum_{i=1}^{\infty} [\rho A s^2 Q_i(s) + EI \alpha \beta_i^4 s Q_i(s) + EI \beta_i^4 Q_i(s) - a_i H(s) + \sum_{j=1}^p d_{ji} P_j(s) - \sum_{j=1}^r b_{ji} F_j(s) - \sum_{j=1}^r e_{ji} M_j(s)] \varphi_i(x) = 0 \quad (3)$$

In Equation (2) symbols: $Q_i(s)$, $H(s)$, $P_j(s)$, $F_j(s)$, $M_j(s)$ denote the Laplace transforms of the time-dependent generalized co-ordinates, excitation forces, and forces and torques originated from the dampers. The other symbols denote coefficients dependent on the base functions. The following form of the distributed loading is assumed: $q(x, t) = h(t)g(x)$.

A linear independence of $\varphi_i(x)$ yields a system of algebraic equations to determine the unknown beam deflection and slope at the points where mass dampers are attached.

These deflections and slopes are given in the s -domain. Having furtherly given the transforms of forces $F_j(s)$ and torques $M_j(s)$, the transforms of the general co-ordinates may be obtained, allowing to calculate from Eqn (2) the transforms of the deflection (and slope) of the beam

$$W(x, s) = \sum_{i=1}^{\infty} Q_i(s) \varphi_i(x) \quad (4)$$

Assuming steady-state vibration, after substituting $s = j\omega$ (where $j = \sqrt{-1}$) the corresponding amplitude-frequency characteristics may be obtained.

3. Random excitation with PSD

Experimental results have shown that many types of random excitations have peaked PSDs [7]. The power spectral density of a force, PSD_F , is defined in [7]

$$PSD_F = \frac{2a\bar{x}}{\sqrt{\pi}} (a\Omega)^2 \exp(-(a\Omega)^2) \quad (5)$$

where

$$\bar{x} = \int_{-\infty}^{+\infty} PSD_F d\Omega, \quad \Omega = \frac{\omega}{\omega_0} \quad (6)$$

and a represents the peak parameter, ω_0 is the frequency to which the dampers are tuned. It is assumed that the beam is subjected to the distributed or concentrated forces with peaked PSDs.

4. Optimization control problems

Two optimization issues are considered. The first is a local type passive problem of vibration minimization, the second is a global type semi-active vibration control problem.

The implementation of the M.J.D. Powell's TOMLIN [8] sequential quadratic programming procedure, included in the IMSLC/Math/Library package was used to solve the optimization problems.

4.1. Passive local vibration control – tuned mass dampers

The objective function is the root mean square (RMS) of the beam deflection at any arbitrary chosen point. The aim of optimization is to find the values of the design variables: damping ratios, frequency ratios and positions of the TMD's, which minimize the objective function. The numbers of translational and rotational dampers and its mass ratios are input parameters for the optimization procedure, the practical constraints were imposed: damping ratios and frequency ratios must lie within the ranges <0,1> and <0,2> respectively.

The problem is well known, but the novelty of the approach is the use of the rotational dampers, which can significantly improve damping efficiency, especially when the translational damper cannot be placed at the point of excitation force.

4.2. Semi-active global vibration control – tunable dynamic vibration absorbers

The objective function is the RMS of the time averaged kinetic energy of a whole or any arbitrary chosen part of the beam. The aim of optimization is to find the positions of the translational and rotational tunable dynamic vibration absorbers: damping is neglected, the absorbers are tuned to be resonant at each excitation frequency. The numbers of

translational and rotational absorbers and its mass ratios are input parameters for the optimization procedure.

This problem may turn out to be very complex, even for a simple structure considered, because a system with tunable dynamic vibration absorbers (vibration neutralizers) is very sensitive to inaccurate tuning and location. Moreover, additional resonant frequencies may appear in the system.

5. Conclusions

The research deals with damping of vibration in beams subjected to random excitations (concentrated and distributed) with peaked PSDs. The vibrations are suppressed by the system of translational and rotational mass dampers attached to a beam.

The numerical algorithm allows to calculate the amplitude-frequency characteristics of the deflection, slope, and bending moment at any point of the beam, and the kinetic energy of the whole beam or its part.

Two optimization issues, passive and semi-active, local and global, with tuned mass dampers and tunable vibration absorbers are presented.

The use of the rotational dampers may significantly improve damping efficiency, and they can work effectively even if mounted in places where the translational ones do not work (for example in the nodes of the vibration mode).

The presented analysis concerns the problems of vibration in beams only, but the rotational mass dampers (and vibration absorbers) can improve damping efficiency also in other structures transferring bending moment, such as frames and plates.

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