Computational Model of the Deformation of Thin Gradient Coating Lying on Nondeformable Foundation

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Abstract

One of the analytical models applicable to non-homogeneous material is presented. This model uses the concept of functionally graded materials (FGM), which are multiphase composites with a continuous change in the concentration of constituents by volume and demonstrate smooth changes in the mechanical properties from point to point. The computational model of the deformation of a thin gradient coating lying on an nondeformable base is a solution of the contact problem for an inhomogeneous thin layer, rigidly fixed to the lower face or lying without friction on the non-deformable base (sliding seal). The model is based on the effective numerical method, namely bilateral asymptotical method, described in papers of S.M. Aizikovich, etc. [1, 2]. The specificity of this method is in representing the kernel transformation for the dual integral equations (DIE) using a special form suitable for analytical solutions. This method makes it possible to obtain a solution of the elasticity modulus problem (Young's modulus, Poisson's ratio, Lame's modules); it is a continuously differentiable function in the coordinate corresponding to the thickness of the coating. The article presents the results of the study of the effect of an arbitrary change in the modulus of elasticity on functional gradient coatings with a complex structure. It is proved that coatings with variable elastic properties have greater resistance to wear and contact stresses This problem is relevant in connection with the need for technological prediction of delamination of modern coatings rigidly adhered to an nondeformable base

Keywords: contact problem; graded properties; inhomogeneous layer; dual integral equations

1. Formulation of Contact Problems for the Functionally Graded Layer and Strip

Static contact problems: the shear by the flat punch $|x|<a$, $-\infty<z<\infty$ of the layer non-homogeneous with depth, rigidly bounded with nondeformable foundation (S1); the indentation by the nondeformable punch with width $2a$ and indenter shape $\gamma(x)$, into the inhomogeneous with depth strip (S2) are discussed. It is proposed the shear force P acts on each unit of punch length in the parallel direction to z axes, so it inspire punch displacement δ along the z, and is the cause of pure shear deformation into the layer (S1) or the indentation force P cause the punch displacement along the x axes into δ value (S2) (Fig. 1).

Poisson's ratio ν, layer shear modulus G, strip Young's modulus E are arbitrary functions of y,

$$E = E_0 f(y),\ G = G_0 f(y),\ \nu = \nu_0 f(y),\ -h \le y \le 0 \qquad (1)$$

where $f(y)$ – continuously differentiable function, not equal to zero for tolerance region, and gradient of elastic properties law into layer may be positive, negative or change its sign with the depth of layer.

We assume frictionless interaction the punch and elastic layer, and region around the punch are free of stresses. It needs to determine the distribution of the contact shear (S1) or normal stresses(S2) under the punch.

$$\tau_{yz}(x,0,z) = -\tau(x),\ \sigma_y(x,0,z) = -q(x),\ |x| \le a, z = 0 \qquad (2)$$

τ_{yz}, σ_y – are the continuous function of coordinates corresponding to shear and normal stresses on the upper surface z = 0 of the layer or strip respectively.

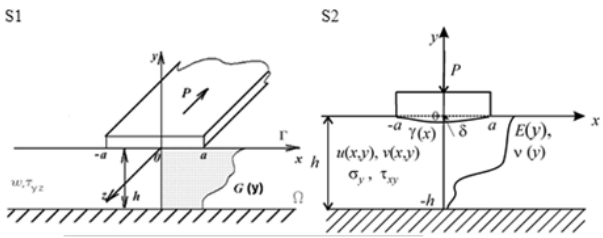

Figure 1: Schemes of contact problems for the inhomogeneous strip

By means of a Fourier transformation, the contact problems can be reduced, in terms of the unknown normal contact stresses beneath the punch $q(x)$ to the Fredgolm integral equation

$$\int_{-1}^{1} \varphi(\xi) k\left(\frac{\xi - x}{\lambda}\right) d\xi = 2\pi C_0 g(x'),\ |x'| \le 1,\ k(t) = \int_0^{\infty} \frac{L(\alpha)}{\alpha} \cos(\alpha t) d\alpha \qquad (3)$$

where $x\_= x/a$, $\lambda = h/a$, $\varphi(\xi) = q(x')/C_0$ – unknown contact stresses function. Here, $g(x') = \varepsilon/a$, $C_0 = G(0)$ (for the S1 case), $g(x') = (-\delta + \gamma(x))/a$, $C_0 = \theta(0) = G(0)/(1-\nu)$ (for the S2 case).

2. The reduction of contact problems for inhomogeneous media to the DIE

The equation of spatial elasticity theory in Cartesian coordinates with substitution Hook's law ratios are represented as system of differential equations with variable coefficient for the functions – transforms of Fourier integral transformation. For the described problems there are the ordinary differential equation of the second order for antiplane problem (S1) or the ordinary differential equation of the fourth order for plane problem (S2). It is convenient to rewrite these systems in vector form:

$$d\vec{\mathbf{w}}(\alpha, y)/dy = \mathbf{A}\vec{\mathbf{w}}(\alpha, y),\ -h \le y \le 0 \qquad (4)$$

Here **A** – the square matrix with varied elements, **w**(α, y)=(W,W′) for the S1 case or **w**(α, y)=(V,V′,U,U′), for the S2 case. The boundary conditions for eqn.(4) are followed from conditions onto up and low layer faces.

We suggest the solution of the vector differential equation (4) in the form of linear combination of vectors of fundamental system of solutions.

$$\vec{\mathbf{w}}(\alpha, y) = \sum_{i=1}^{r} C_i(\alpha)\vec{\mathbf{a}}_i(\alpha, y)\,\mathrm{e}^{\gamma_i y},\ -h \le y \le 0 \qquad (5)$$

Here $\gamma_{1,2} = -\alpha$, $\gamma_{3,4} = \alpha$, r – rank of matrix **A**. It is constructed with the method based on the isolation of the exponential components in Green matrix construction. It is allows getting the stable numerical algorithm.

Vectors $\vec{\mathbf{a}}_i(\alpha, y)$ are changing along y axes slower then exponential function and obtained from the set of Cauchy problems for distinct values α

$$d\vec{\mathbf{a}}_i(\alpha, y)/dy = \mathbf{A}\vec{\mathbf{a}}_i(\alpha, y) - \gamma_i\vec{\mathbf{a}}_i(\alpha, y),\ -h \le y \le 0,\ i = 1,\dots,r \qquad (6)$$

The initial conditions for eqn. (6) are obtained from the appropriate solution for homogeneous medium cases onto low face of the layer $y = -h$. So, the next expressions were obtained for numerical construction of DIE transform functions $L(\alpha)$

$$\text{S1: } L(\alpha) = |\alpha|\left(a_1^1 - a_2^1\,\mathrm{e}^{-2|\alpha|h}\right)\left(a_1^2 - a_2^2\,\mathrm{e}^{-2|\alpha|h}\right)^{-1},$$

$$\text{S2: } L(\alpha) = \sum_{i=1}^{4} C_i(\alpha)a_i^3,\ a_i^j = a_i^j(\alpha, 0) \qquad (7)$$

3. Equations Solution of DIE of the problems

3.1. Approximation of the kernel transforms functions for DIE

For the considered problems it can be demonstrated similar [3] the kernel transform $L(a)$ of DIE has the following properties under the preceding hypothesis the elastic modules are neither vanish no become infinite.

$$L(\alpha) = A|\alpha| + B\alpha^2 + \mathrm{O}(\alpha^3),\ \alpha \to 0;\ A = \lim_{\alpha\to 0} L(\alpha)\alpha^{-1};$$

$$L(\alpha) = 1 + C|\alpha|^{-1} + D\alpha^{-2} + \mathrm{O}(\alpha^{-3}),\ \alpha \to \infty; \qquad (8)$$

where A, B, C, D are some constants. The properties of $L(\alpha)$ for the graded layer for the small α (α→0) are differed essentially from the laminated media. The theorem in [3] proves if function $L(\alpha)$ has the properties (7) it can be approximated by the special form:

$$L(\alpha) = L_N(\alpha) = \mathrm{th}\,A\alpha \prod_{n=1}^{N}\left(\alpha^2 + \delta_n'^2\right)\left(\alpha^2 + \gamma_n'^2\right)^{-1} \qquad (9)$$

For a numerical realization the improving approximation of $L(\alpha)$ by functions in eqn. (9) approximation coefficients δ_n, γ_n can be estimated by using the algorithm described in [1].

3.2. Solution of DIE of the problems

So, Fredgolm integral equation (3) is reduced to the dual integral equation whose solution for the function $L(\alpha)$ of the form (9) is presented in [2] in the case when the function $g(x')$ can be represented as a Fourier series and expressed in terms of associated Legendre functions and the integrals of them.

It's proved that the solutions obtained are asymptotically accurate, both for small and large values of the dimensionless geometric parameter λ simultaneously. Comparison of the obtained closed form solution with the known solution for a homogeneous band obtained by Vorovich I.I. at all, displays an error of less than 1 p.c. for λ <1 and λ> 2.8. The ultimate error (more than 10 pc) is observed at λ = 2.

In a numerical study of how the approximation accuracy of the kernel transformation function with the expression (5) affects the validity of contact stresses, it should be recognized that for contact problems for a homogeneous medium similar to monotone laws of elastic properties $f(y)$ for the inhomogeneous medium it's sufficient to approximate L(α) just one multiplicand in eqn. (9) ($N = 1$). Accordingly, approximation error is less than 3% on the average. Thus the best approximation for the S2 problem is $L_2(\alpha)$. The laws of inhomogeneity $f(y)$, whose derivatives repeatedly change sign along the thickness of the layer (which corresponds to the complex structure of the medium), cause a greater number of multiplicities in eqn. (9) to approximate the kernel transformation function for the integral equation.

The effectiveness of this method is illustrated by the example of contact problem S1 for the case one character set of inhomogeneity laws modeling graded properties of the layer. The shear modulus has the form:

$$G(y) = G_0\,fc(y),\ fc(y) = 1 + (G_0/G_{\min} - 1)\sin^2(c\pi y),\ \nu = 1/3,\ -h \le y \le 0$$

The laws set fc correspond to the case when the soft layer is over of more rigid ones.

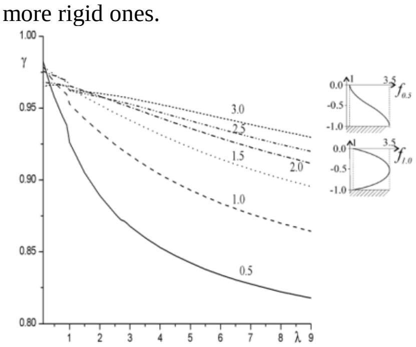

Figure 2: Relation γ(λ)=P(λ)/P_0(λ) between shear force and punch displacement for the fc inhomogeneity kind relating to the homogeneous one for the different values of C_k={0.5,1.,…,3.0}.

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