An improved ground structure method with adjusted displacements in empty zones

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Abstract

The paper deals with an improved method of solving large-scale linear programming problems related to Michell trusses. As in the previous version (reported on CMM 2015), both bars and nodes are activated or deactivated in subsequent iterations. In the present version of the method, the adjoint displacements of inactive nodes, i.e. nodes in empty regions, are specifically adjusted by solving an auxiliary optimization problem, just to reduce the size of the optimization problem. This additional step is cheap and allows a more efficient cutting off the design domain, thus enabling a significant reduction of the size of the problem. Therefore, the numerical results can be attained for denser ground structures than before, giving better approximations of Michell structures. This is very important for 3D problems, because every unnecessary node can generate a large number of unnecessary potential bars. The preliminary results of such problems clearly indicate high efficiency of the proposed method.

Keywords: structural topology optimization, Michell structures, adaptive ground structure method, linear programming, interior point and active set method

1. Introduction

The main advantage of ground structure methods consists in possibility of formulating the topology optimization problem in linear programming forms (i.e. convex and free of local minima), thus enabling to find the globally optimal solution. Nevertheless, a dense set of nodes with a large number of potential bars is required to obtain a good approximation of an exact solution, representing Michell truss. This inevitably leads to large-scale optimization problems which can, however, be solved effectively by using the adaptive ground structure methods (c.f. the papers by Gilbert and Tyas [1], Pritchard et al. [5] and Sokół [6,7]).

In the present paper, an important improvement to adaptive ground structure methods is proposed. This improvement allows better reduction of the problem size by eliminating greater number of unnecessary bars. This reduction is possible thanks to adjusting adjoint displacement field in empty regions where no material is needed. This adjusting phase requires solving an auxiliary linear programming problem of relatively small size. This improvement is particularly important for 3D problems because the optimal spatial trusses tend to assume forms of lattice surfaces while most of design space becomes empty.

2. Fundamentals of plastic layout optimization

Due to limited space of the abstract, we discuss bellow only formulations for trusses (ground structures) subjected to a single load condition. The multi-load case problems will be discussed in the full-length paper (c.f. [5-7]).

2.1. Primal and dual forms of truss topology optimization

The primal (lower bound) form of the linear programming problem defining the lightest truss composed of M bars and working in permissible stresses in tension and compression ($-\sigma_C \le \sigma_i \le \sigma_T$, for $i = 1, 2, \ldots, M$) can be written as follows (for alternative formulations see [1,5-7]):

$$\begin{aligned} \min_{\mathbf{v}^+,\mathbf{v}^-\in\mathbf{R}^M} \quad & V = \sum_{i=1}^{M} v_i^+ + v_i^- = \mathbf{1}^T\mathbf{v}^+ + \mathbf{1}^T\mathbf{v}^- \\ s.t. \quad & \sigma_T \mathbf{C}^T\mathbf{v}^+ - \sigma_C \mathbf{C}^T\mathbf{v}^- = \mathbf{P} \\ & \mathbf{v}^+ \ge \mathbf{0}, \quad \mathbf{v}^- \ge \mathbf{0} \end{aligned} \tag{1}$$

where V is the total volume of the structural material of the truss; $\mathbf{v}^+$ and $\mathbf{v}^-$ are vectors of member volumes of bars in tension and compression, respectively; $\mathbf{1}$ is the vector of length M whose all elements are equal to 1; $\mathbf{P}$ is the vector of nodal forces (of size N - the number of degrees of freedom); and the rectangular matrix $\mathbf{C}_{N\times M} = \mathbf{B}/\mathbf{L}$ is calculated from the geometric matrix $\mathbf{B}$ and the vector of bar lengths $\mathbf{L}$ (by dividing every i-th row of $\mathbf{B}$ by the corresponding length L_i).

In the adaptive ground structure methods, the problem (1) is solved iteratively for some specifically chosen subsets of active bars. Thus, we replace one large-scale optimization problem by a series of much smaller sub-problems with only m active bars, where $m << M$ is several times smaller than M. Decision of activating (or deactivating) new bars is performed based on the dual (upper bound) form, which is given by

$$\begin{aligned} \max_{\mathbf{u}\in\mathbf{R}^N} \; & W = \mathbf{P}^T\mathbf{u} \\ s.t. \quad & \sigma_T \mathbf{C}\,\mathbf{u} \le \mathbf{1} \\ & -\sigma_C \mathbf{C}\,\mathbf{u} \le \mathbf{1} \end{aligned} \tag{2}$$

Here $\mathbf{u}$ is the vector of Lagrange multipliers, called *adjoint nodal displacements*; and W is the work of external loads over these displacements.

Note that constraints $(2)_{2,3}$ correspond strictly to Michell's optimality criteria. It means that optimal truss has to satisfy all of these constraints, i.e. for every potential bar. Moreover, if any constraint is violated, then by activating the corresponding bar (and solving the problem again) we can only decrease W_{max}, and at the same time find better V_{min}, because from duality principle $W_{max} = V_{min}$. It holds for feasible problems (1) and (2), but this is the case considered in the paper (our ground structures are connected properly).

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It is worth noting also that dual variables **u** are calculated automatically and do not require additional computations if one uses the *primal-dual* version of the *interior point method*.

3. The adaptive ground structure method with selective subsets of active bars and nodes

At the beginning of this section, it should be reminded that the adjoint displacement and strain fields in empty regions are not defined uniquely. Consequently, one can adjust the non-unique displacement field just for a purpose, which in our case is the maximal possible to attain reduction of the size of the optimization problem.

In the prior version of the method developed by the author [7], the displacements of nodes in empty regions were assumed equal to the displacements from preceding iterations (see point 8 of the algorithm in Sec. 3 of [7]), and this assumption was justified. Nevertheless, on the base of the performed numerical tests, it has been noted that many unnecessary bars are still activated, even if they are absent in the final optimal solution. Consequently, the reduction of the size of the optimization problem of succeeding iteration was not as significant as expected. The present, updated version of the method eliminates this drawback. Instead of utilizing the values of nodal displacements in empty regions from the previous iteration, it is worth to solve an auxiliary optimization problem to adjust displacement field in a better way.

3.1. Auxiliary optimization problem to adjust nodal displacements in empty regions

The aim of the present investigation is to develop a procedure of adjusting the displacement field in empty regions in such a way that the number of new activated bars in the subsequent iteration is as small as possible. Since activation of new bars corresponds to violated constraints $(2)_{2,3}$, it is justified to minimize the average normalized strain over m_β inactive bars of first level connectivity

$$\begin{aligned} &\min_{\substack{\boldsymbol{\varepsilon}\in\mathbf{R}^{m_\beta}\\ \mathbf{u}^\beta\in\mathbf{R}^{n_\beta}}} \quad \varepsilon_{avg} = \frac{1}{m_\beta}\sum_{i=1}^{m_\beta}\varepsilon_i \\ &s.t.\quad \left.\begin{aligned} \varepsilon_i &\geq \sigma_T\left(\mathbf{C}_i^\alpha\mathbf{u}^\alpha + \mathbf{C}_i^\beta\mathbf{u}^\beta\right)\\ \varepsilon_i &\geq -\sigma_C\left(\mathbf{C}_i^\alpha\mathbf{u}^\alpha + \mathbf{C}_i^\beta\mathbf{u}^\beta\right)\end{aligned}\right\}\ \text{for } i = 1:m_\beta \end{aligned} \tag{3}$$

Here ε_i denotes normalized adjoint strain of *i-th* inactive bar; $\mathbf{u}^\alpha$ and $\mathbf{u}^\beta$ are vectors of displacements of active and inactive nodes, respectively. Note that $\mathbf{u}^\alpha$ is known and fixed, and only displacements $\mathbf{u}^\beta$ and $\boldsymbol{\varepsilon}$ play the role of design variables. Moreover, $\mathbf{C}_i$ denotes the i-th row of matrix $\mathbf{C}$ which is divided here into two parts $\mathbf{C}^\alpha$ and $\mathbf{C}^\beta$ corresponding to active and inactive degrees of freedom.

Alternative strategies of minimizing strains in empty regions will be discussed in the full-length paper.

4. Example - Michell sphere

Consider a well known problem of Michell sphere, that is the minimum weight 3D structure to support a pair of axial torques [2-4]. Using antisymmetric boundary conditions, the problem can be reduced to one eighth of the full cuboidal domain surrounding the sphere. The result presented in Fig. 1 has been obtained for such a reduced problem. For the sake of clarity, only the upper half of the sphere without loading and supports is shown. The base of this visible part of the sphere is fully supported while the torque on the upper side is modelled by 24 point loads distributed circumferentially every 15 degree. These point loads are applied on a circle of radius $r = 3$, located by $h = 4$ above the origin; and as was expected, the optimal layout forms a latticed sphere of radius R = 5.

The numerical result presented in Fig. 1 was obtained using a ground structure of 63×63×48 cubic cells with 16 753 176 554 bars. The solution was found in 9 iterations with total CPU time less then 7 hours and final volume $V_n = 4.42509\mathrm{M}/\sigma_0$. The numerical result is worse than the exact result of only 0.7% ($V_a = 4.394449155\mathrm{M}/\sigma_0$, see [3]). More details of the present example as well as other numerical solutions of 2D and 3D Michell problems will be discussed in the full-length paper.

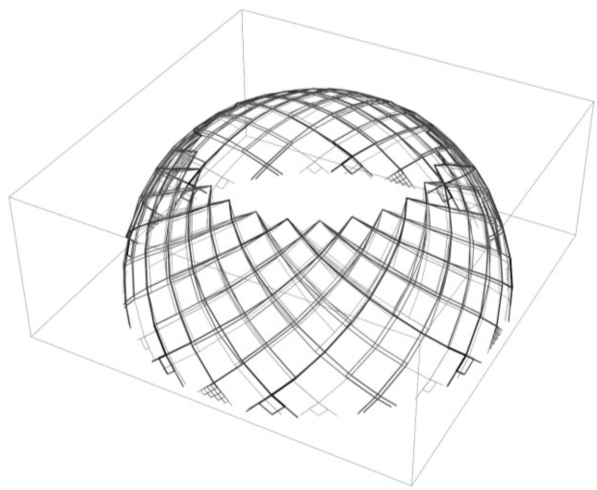

Figure 1: Numerical approximation of Michell sphere using a ground structure of almost 6 billion bars

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