# Synchronisation phenomenon in the de-tuned three blades rotor driven by regular or chaotic oscillations 

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#### Abstract

The aim of the paper is to analyse the synchronisation phenomenon of a rotating structure composed of three beams attached to a rigid hub. It is assumed in the analysis that one beam is $10 \%$ thicker comparing to the remaining ones. Furthermore, two possible variants of excitation are considered: (a) torque given by harmonic function or (b)torque produced by a chaotic oscillator. Moreover, non-classical effects as material anisotropy, transverse shear and cross-section warping are taken into account for flexural composite beams oscillations. The Hamilton's principle is used to derived the partial differential equations of motion. The reduction to ordinary differential equations of motion have been done by Galerkin's method. Next, the equations have been solved numerically and the resonance curves have been analysed in terms of synchronised motion of the hub and blades of the rotor.


Keywords: rotating structures, composite beams, beam synchronisation, chaotic motion

## 1. Introduction

The famous Huygens experiment with synchronisation of two clocks have been recalled in [2], where the basic definition of synchronisation phenomenon has been given. Most of the papers about synchronisation phenomenon described structures composed of pendulums in a typical vertical plane. However, the synchronisation of the hub with two pendulums rotating in a horizontal plane have been presented in [5]. Furthermore, authors in that paper analysed nonlinear dynamic of the rotor. The rotating structures are very well known in mechanical engineering, standard examples are wind turbins, jet engines and helicopter rotors. Dynamics of a thin-walled composite rotating structure in linear rigor have been studies in [1, 3].

## 2. Model and equations of motion

The paper investigates a system composed of three composite elastic beams attached to a rigid hub of radius $R_{0}$, as shown in Figure 1.


Figure 1: Model of the rotating hub with three elastic beams
The hub may oscillate or rotate about vertical axis $C Z_{0}$, the current position is given by an angle $\psi$, the length of each beam is describe by $l_{i}$, the width and height of the cross-section by $d_{i}$ and $h_{i}$, respectively, where $i=1,2,3$. The system considers two identical beams (No. 1 and 3) and one with $10 \%$ higher thickness thus, the second (No. 2) beam is de-tuned. The composite material of the beams is linearly elastic.

The Hamilton's principle of least action was used to derive
the system of the partial differential equations (PDEs) for the hub and for the each beam $[1,3]$. Next, the reduction from the PDEs to ordinary differential equation was done by the Galerkin projection method. After conversion, the set of governing equations takes the form:

$$
\begin{array}{r}
\left(1+J_{h}+J_{b 2}+J_{b 3}-\alpha_{h 12} q_{1}^{2}-\alpha_{h 22} q_{2}^{2}-\alpha_{h 32} q_{3}^{2}\right) \dot{\Omega} \\
+\alpha_{h 11} \ddot{q}_{1}+\alpha_{h 21} \ddot{q}_{2}+\alpha_{h 31} \ddot{q}_{3}-\alpha_{h 13} q_{1} \dot{q}_{1} \Omega \\
-\alpha_{h 23} q_{2} \dot{q}_{2} \Omega-\alpha_{h 33} q_{3} \dot{q}_{3} \Omega+\zeta_{h} \Omega=\mu \\
\ddot{q}_{1}+\alpha_{12} \dot{\Omega}-\alpha_{14} q_{1} \dot{q}_{1} \Omega+\left(\alpha_{11}+\alpha_{13} \Omega^{2}\right) q_{1}+\zeta_{1} \dot{q}_{1}=0 \\
\ddot{q}_{2}+\alpha_{22} \dot{\Omega}-\alpha_{24} q_{2} \dot{q}_{2} \Omega+\left(\alpha_{21}+\alpha_{23} \Omega^{2}\right) q_{2}+\zeta_{2} \dot{q}_{2}=0 \\
\ddot{q}_{3}+\alpha_{32} \dot{\Omega}-\alpha_{34} q_{3} \dot{q}_{3} \Omega+\left(\alpha_{31}+\alpha_{33} \Omega^{2}\right) q_{3}+\zeta_{3} \dot{q}_{3}=0 \tag{1}
\end{array}
$$

Dynamics of the hub is described by the first equation of the set of Eqs.(1). The $J_{h}$ and $J_{b i}$ are dimensionless mass moment of inertia of the hub and dimensionless mass moment of inertia of each beam, respectively expressed as a magnitude of the inertia of the first beam. The dimensionless angular velocity is described by $\Omega=\frac{d \psi}{d t}$. Excitation is defined by torque $\mu$ imposed on the hub, and is considered in two variants: (a) as a driving torque, expressed as $\mu=\mu_{0}+\rho \sin \omega t$ or (b) as a chaotic oscillator, where $\mu=\rho x$ and $x$ is calculated from the Duffing's equation $\ddot{x}+k \dot{x}+x^{3}=B \cos \omega t$, that was presented in [4]. The dynamics of the beams is described by the last three equations of set (1), where $q_{1}, q_{2}$ and $q_{3}$ are generalized coordinates corresponding to the complex flexural-torsional specimen deformation. Damping of the system is included by arbitrary introduced viscous damping coefficients $\zeta_{1}, \zeta_{2}, \zeta_{3}$ and $\zeta_{h}$ for the beams and hub, respectively.

## 3. Numerical studies

Numerical studies have been performed for the system composed of three beams attached to the rigid hub, considering that beam No. 1 and No. 3 are identical, but beam No. 2 is de-tuned, due to $10 \%$ higher thickness. All $\alpha_{i j}$ factors present in equations of motion (1) have been derived for the physical model and are given in table 1.

[^0]Table 1: Dimensionless coefficient, based on the real physical model

| $\alpha_{11}=\alpha_{31}$ | 12.0388808306 | $\alpha_{21}$ | 14.5664355288 | $\alpha_{h 11}=\alpha_{h 31}$ | 0.0112028592 | $\alpha_{h 21}$ | 0.0123216629 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha_{12}=\alpha_{32}$ | 1.9765664564 | $\alpha_{22}$ | 1.9768127528 | $\alpha_{h 12}=\alpha_{h 32}$ | 0.0084706516 | $\alpha_{h 22}$ | 0.0093152417 |
| $\alpha_{13}=\alpha_{33}$ | 0.4946581624 | $\alpha_{23}$ | 0.4946272964 | $\alpha_{h 13}=\alpha_{h 33}$ | 0.0169413032 | $\alpha_{h 23}$ | 0.0186304834 |
| $\alpha_{14}=\alpha_{34}$ | 1.55170796 | $\alpha_{24}$ | 1.551483824 | $\zeta_{h}$ | 0.1 | $\zeta_{i}$ | 0.002 |

As it has been mention earlier, rotor dynamics is studied for two variants when driven by harmonic torque or the chaotic oscillator. However, in this abstract we will present only solutions for periodic driving torque.


Figure 2: Resonance curve of angular velocity of the hub for amplitude of excitation $\rho=0.01$

Numerical simulations have been performed to the arbitrary selected amplitude of excitation $\rho=0.01$ of the harmonic function. Response of the hub is shown in the Figure 2, with two resonances, first close to $\omega \approx 3.48$ and second $\omega \approx 3.82$.


Figure 3: Resonance curves of displacements of each beam (beam No. 1 and No.3- green line, beam No. 2 - blue line for amplitude of excitation $\rho=0.01$.

In the Figure 3, response of the beam is shown, green line represents two identical beams (No. 1 and No.3), while blue line concerns the response of the de-tuned beam. In both cases two resonances occurred, first close to $\omega \approx 3.48$ and the second one close to $\omega \approx 3.82$. The main resonance is observed for beams No. 1 and No.3, with very small oscillations of the second beam. In this frequency zone we observe synchronisation of motion of beams No. 1 and No. 3 and large reduction of motion of beam No.2. Around the second resonance zone oscillations are localised in beam No.2, while beams No. 1 and No. 3 are very small but still synchronised.


Figure 4: Portrait of a strange chaotic attractor obtained from Duffing equation, applied as driving torque.

In the future studies will consider de-tuned structure excited by the chaotic oscillator. The Poincaré portrait of the chaotic oscillator, which will be used to excite the system is shown in the Figure 4.

## 4. Conclusion

For the detuned structure two resonances have been observed, the fist resonance occurs close to $\omega \approx 3.48$ and the second at $\omega \approx 3.82$. Resonance curves for amplitudes of the first and third beam are exactly the same $\left(q_{1}, q_{3}\right)$, due to their full symmetry. From the other hand, the main resonance for the beam No. 2 is very small. For the second resonance zone, vibrations are localised in the beam No. 2 with very small oscillations of beams No. 1 and 3. At the first and the second resonance identical beams No. 1 and 3 exhibit the complete synchronisation of motion. For the second beam synchronisation with locked phase is noticed.

## References

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