Qualitative analysis of properties of numerical approximation of an inverse relation by suitably trained Artificial Neural Network for two examples of inverse solutions

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Abstract

If the inverse problem is solved by numerical approximation of an unknown inverse relation (by a suitable ANN that is chosen to approximate the inverse relation), the mathematical formulation of the inverse problem is absent. Only forward problem is solved many times to prepare the training set for ANN. Thus a formal analysis of mathematical formulation of inverse problem is impossible. In this paper it is shown (by examples) that this missing analysis can be substituted by observations of the numerical properties of the training process (progress and results). The subject of the paper is an exemplification of the facts that: if the solution of the inverse problem does not exists or if the solution of the inverse problem is not univoque (ambiguous) or if the solution of the inverse problem is not well conditioned by the experimental data, an analysis of the learning process of the ANN in the numerical model of the inverse problem allows to discover this without any prior qualitative analysis. These statements are illustrated by two examples: FWD test for discovering internal structure of a multi-layered plate and identification of parameters in adsorption problem.

Keywords: inverse problem, approximation of inverse relation by ANN, Artificial Neural Network, FWD test

1. Introduction

Some of the most important and well-studied mathematical issues belong to inverse problem category. In general terms, an *inverse* problem is a generic context of investigation in which information on a physical quantity, or more generally on a system, are sought starting from measurements or information of the indirect type. It is the inverse of a *forward* problem, in which the effects are computed starting from their causes.

In paper [1] and in same conference papers (f.ex. [2]) numerical solutions of various inverse problems have been presented. The forward problem was always defined by a set of differential equations. The forward solution was thus any of engineering solution that determinates the unknown fields of these equations having given material parameters and loads. Roughly speaking, the inverse problem is the problem of identification of material parameters and loads for which independent variables of the problem, measured in some points of the material domain, take given, known values.

The kernel of the method presented in our former papers is an explicit approximation of an inverse relation, assumed always in a form of an Artificial Neural Network (ANN). The inverse relation attributes trial material parameters or loads (appearing at the output of the ANN) to the computed via forward solution, “measured” data (put at the input of the ANN). In order to obtain the correct approximation we have used the same algorithm of the network training for many various problems. It is clear that the formal, mathematical definition of the inverse problem is not necessary in this approach. To construct the inverse relation it is enough to repeat many times solution of the forward problem, to build a set of examples for ANN training. However, the formal analysis of an inverse formulation is very important since inverse problems are often not well posed. Typically the sensitivity of the inverse solution on the type and quality of the input (“measured”) data is high. More, some material parameters can be difficult to be identified from the given data. This formal analysis of mathematical formulation of inverse problem is impossible in the frame of our approach.

In this paper it is shown (by examples) that this missing analysis can be substituted by observations of the numerical properties of the training process (progress and results).

2. FWD test as an example of different sensitivity of various material parameters on the collected data

The Falling Weight Deflectometer (FWD) is an instrument for “in situ” tests, commonly used to evaluate mechanical parameters and to assess the quality of layered structures of road and airfield pavements. The deflection, response of the pavement to an applied load is used here as an indicator of material properties, and structural performance of the pavement. Deflections due to an impulse load from the falling weight are measured in several points (6 to 9) using geophones aligned on a rigid support, mounted on a special purpose vehicle. The theoretical deflection is computed basing on the theory of elastic, layered half-space statically loaded over the area of the falling mass (forward problem). We use virtually simulated data for the ANN training. We consider two numerical models: the above mentioned elastic, layered half-space and layered plate on the Winkler elastic subgrade. For both models the Young moduli and Poisson ratios for three layers or for two layers plus a Winkler stiffness of the subgrade have been identified. The structure of the biggest ANN was taken as follows ({content of the layer}): {six input nodes valued with the measured deflections plus three nodes for the layers thicknesses}; {variable number of hidden nodes}; {chosen mechanical parameter}. This is thus simply a system of as many networks as needed, all trained with a classical BP algorithm, all of them – fully interconnected. We show how that the learning for identification of the properties of thin layer (that are not well conditioned by the FED data) progress. The training is very slow in this case. In contrast, it is shown that test set error and learning set error for identifications of dominating layers are very small and the learning is several times quicker. We analyse also the properties of the training for identification of the stiffness of the subgrade. The example of comparison is illustrated in Figure 1. Finally we analyse a case of non univoque situation (a pathological combination of material parameters), we propose an algorithm of the solution in this case. It is possible to discover that fact using small ANN. It exists an algorithm of splitting the problem into a set of two or more univoque problems.

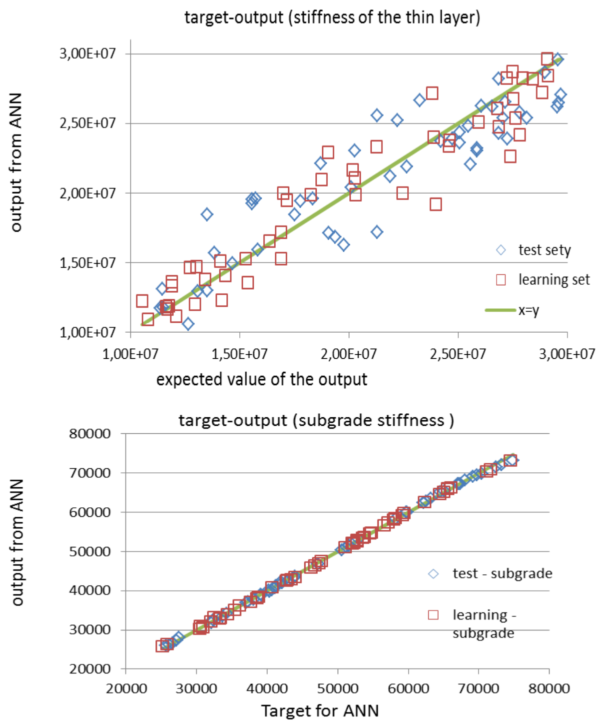

Figure 1: comparison of learning quality for two parameters of the struture

3. Identification of parameters of adsorption process as an example of training behaviour in a case of wrong formulation of the problem

In an equation of dispersion with adsorption (1) we have to identify parameters of the adsorption models appearing in eq. (2) ("retardation factors" of Langmuir and Freundlich respectively that govern the sorption S). Source intensity Q and its position, conductivity k and diffusivity D of the porous medium are given in this example. The concentration field *c* and average velocity *v* of a flow across a permeable stratum are the forward solutions here. Obviously, concentrations *c* are measured and given in few point in some time instances.

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial c}{\partial x_j}\right)-\frac{\partial}{\partial x_i}\left(v_i c\right)-\frac{\partial S}{\partial t}+Q \qquad (1)$$

$$\delta_L\left(c\right)=1+a_m\frac{K}{\left(1+Kc\right)^2} \qquad \delta_F\left(c\right)=1+a_m K\beta_d c^{\beta-1} \qquad (2)$$

In examples, the considerations are limited to the one dimensional case of dispersion with adsorption. It is seen that in the case of Langmuir model identification of a_m and K is possible while in the case of Freundlich model is not. It is shown that this fact is manifesting in such a manner that the training does not progress at all (Figure 2). Also in the case of this example we compare few sets of input (measured) data that are more or less useful for identification of the sorption parameters. This is very important observation since the analysis of the forward solution, (necessary for approximation of inverse relation) can be used, by a way, as a tool of design of the measurement strategy.

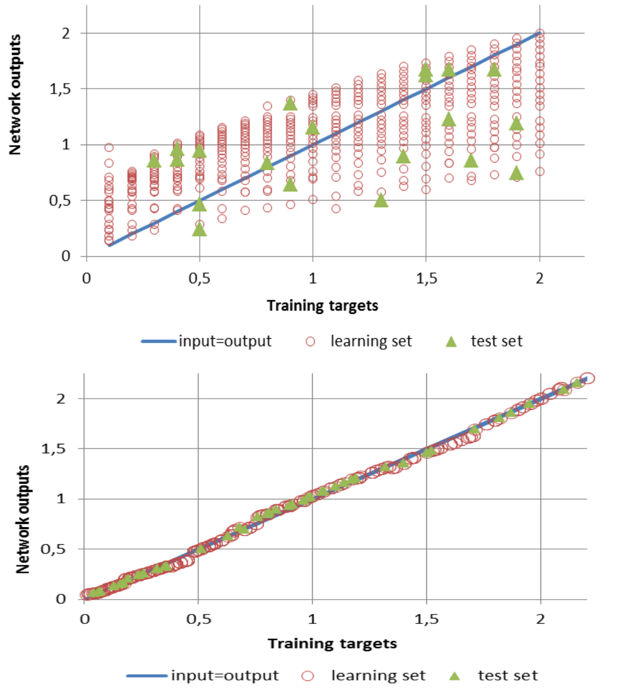

Figure 2: results of the training: upper graph for K in the Freundlich model, bottom graph – identification of the product a_m and K for the same model

4. Conclusions

If the solution of the inverse problem does not exists, an analysis of the learning process of the ANN in the numerical model of the inverse problem allows to discover this without any prior qualitative analysis. In this case the training process does not progress; If the solution of the inverse problem is not univoque (ambiguous), an analysis of the learning process of the ANN in the numerical model of the inverse problem allows to discover this without any prior qualitative analysis; If the solution of the inverse problem is not well conditioned by the experimental data, an analysis of the learning process of the ANN in the numerical model of the inverse problem allows to discover this without any a priori qualitative analysis.

References

[1] Lefik, M., Boso, D., *Inverse problem – soft solution*, Bytes and Science, G. Zavarise and D.P. Boso (Eds.) © CIMNE, Barcelona, 2012.

[2] Lefik, M., Boso, D., Identification of parameters of adsorption by approximation of inverse relation and using artificial neural networks. *http://solmech2016.ippt.pan.pl /Abstracts/0177.pdf*, 40th Solid Mechanics Conference SOLMECH 2016, Warsaw, Poland.

[3] Meier, R.W., *Backcalculation of Flexible Pavement Moduli from Falling Weight Deflectometer Data Using Artificial Neural Networks.* Reports for U.S. Army Corps of Engineers, technical report GL-95-3, Washington DC, 1995.