Experimental study and numerical optimization of tensegrity dome models

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Abstract

The paper deals with the design, experimental analysis and numerical optimization of tensegrity dome models. Two structures are analysed – a Geiger system dome, with PVC-U bars and PA6/PP/PET tendons and a Fuller system dome, with wooden bars and steel cables. All materials are tested in terms of Young's modulus and yield stress values, the bars are also tested for the limit length demarcating the elastic buckling from plastic failure. The obtained data is implemented in SOFiSTiK commercial software FE model. The model parameters are considered uniform random variables. Geometrically and materially nonlinear analysis is carried out. Based on the obtained structural response, a Monte Carlo simulation – based approach is incorporated for both structural design point formulation and the SLS random analysis. Finally, an attempt is made to erect the Fuller dome model in order to compare experimental values and numerical results with the measurements of an actual structure.

Keywords: tensegrity domes, experimental verification, sensitivity analysis, numerical optimization, Monte Carlo methodology

1. Introduction

Tensegrity (tensional integrity) structures were proposed by Richard Buckminster Fuller and Kenneth Snelson in early 1950's. Their main feature is the internal stabilization of the elements loaded with pure axial forces only – tension (tendons) or compression (bars). This means the structure fails only if the tendons yield or the bars buckle.

The advantage of this type of structures lies in the possibility of roofing a large area using a small amount of materials, hence gaining a lightweight and slender structure. The architectural form of the tensegrity structures is also one of its main advantages.

2. Domes geometry and materials used

Two tensegrity domes were designed – both suspended on a wooden support ring of a 6 m diameter. The first structure was a Geiger system dome, the latter – a Fuller system dome. Illustrative sketches of spatial layout of these typical solutions are shown in Fig. 1.

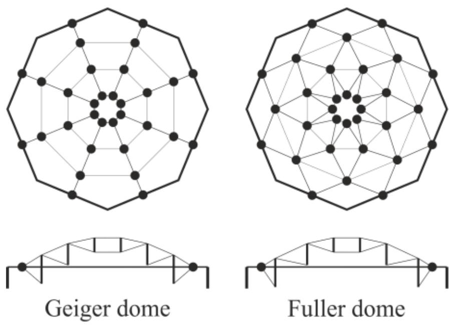

Figure 1: Illustrative sketches of spatial layout of tensegrity systems applied in the work.

In the first case (Geiger dome) the bars were made of PVC-U pipes, while the tendons were made of nylon technical line (PA6/PP/PET monofilament). Synthetic polymers are rarely used in construction as load-bearing elements, hence their suitability for structural design required primary experimental tests. The results were not only satisfactory, the observation of the non-classic material behavior successfully became the basis for the optimization procedures.

In the second case (Fuller dome) the bars were made of structural timber and the tendons were made of S30400 stainless steel 6×7 cables, with a diameter of 2 mm.

3. Experimental study of applied materials

The experimental tests of the materials applied in the study were conducted on the Zwick/Roell Z 400 testing machine, fulfilling the respective PN-EN-ISO standards guidelines on performing strength tests of steel, timber and plastic. For each of the material-configuration variant, 20 samples were tested for Young's modulus and yield stress values.

The nylon monofilament and the steel cable were subjected to a static tensile test in two variants. At first, the materials were put in the grips of the testing machine directly, however the construction method of the real – life nodes forced to analyze the samples also with snap hooks attached. In both variants the samples failed mostly in the vicinity of the anchorage, giving the yield stress values similar in case of direct and indirect anchoring. Similar yield stress values were also obtained if the rupture occurred in the center of the sample.

The PVC-U pipes and the structural timber bar members were subjected to a static compression test. The elastic modulus was investigated, along with the observation of the failure mechanism. Both the rigid and pinned supports were considered for the samples. Also, the samples were examined assuming their length variation, in order to obtain the limit length of the bar demarcating the elastic buckling from plastic failure.

The studies showed that in the case of the applied materials, the stability loss of the bars is a major threat, leaving the plastic tendon failure behind. Also, the experiments showed that fastening the tendons using snap hooks is not effective, as the slip in the knot is non-negligible and may result in improper elastic modulus assessment.

The results of the studies were used in the formulation of the numerical model used for parametric analysis.

4. Numerical study of the geometry – sensitivity analysis and numerical optimization

Both variants of the structure were also analyzed in the SOFiSTiK commercial FEM software. The support ring was modelled with 12-dof beam elements, the bars were modelled as 6-dof truss elements and the tendons as 6-dof cable elements. Nylon tendons and PVC-U pipes were described by bilinear elastic-plastic material law, predefined in SOFiSTiK software, the parameters of their material model were derived from both aforementioned experimental tests and manufacturers' designations. For timber bars and steel cables a standard linear elastic-plastic model was proposed with standard parameters.

Dead load, climatic loads (wind and temperature) and impulse excitation were taken into account. Geometrically and materially nonlinear analysis was conducted, in accordance with TH3 theory, implemented in SOFiSTiK. Additionally, verification of the FEM model was performed to check if the bars were only axially compressed and the tendons axially elongated in every step of the nonlinear numerical analysis.

To ensure the possibility of carrying out numerical calculations, the truss elements had to be supported on spring elements of an infinitesimal elasticity coefficient, hence the support forces of the elements are also infinitely small, consistent with real conditions. The wooden frame was modelled as simply supported, all joints were assumed hinges.

The visualization of the Fuller dome FEM model in SOFiSTiK software is shown in Fig. 2.

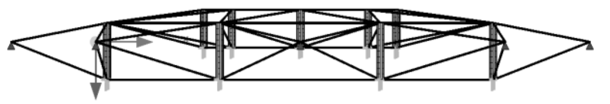

Figure 2: The visualization of the Fuller dome FEM model [2].

Prior to the target optimization process, sensitivity analysis of the initial construction of the Fuller system tensegrity dome has been carried out. The analysis was planned to identify the design variables decisive for the tensegrity system structural response. Primarily, the impact of variations of these parameters on the distribution of internal forces has been taken into account, secondly their impact on the vertical displacements was also investigated.

Sensitivity analysis was performed incorporating the variables for the elastic moduli change, the variation of cross-sectional areas, the length diversity of the elements, cable elements prestressing force detention, dome nodes elevation imperfections and change in the relations between radii forming the structure, among many others.

Sensitivity analysis results for the vertical displacement of the central node of the dome [mm] versus the change for five discrete realizations of five chosen variables (the change in [%]) is shown in Fig. 3.

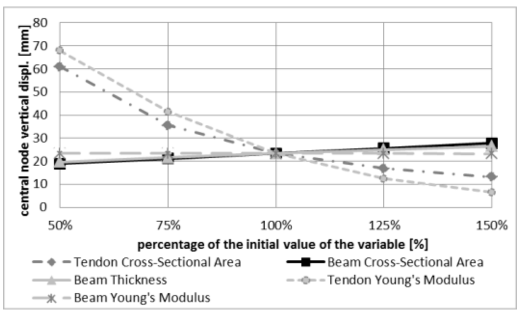

Figure 3: Sensitivity analysis for the vertical displacement of dome's central node versus the change in five chosen variables.

To discern the optimal geometry, the crude Monte Carlo simulation methodology was employed, incorporating the variance reduction techniques [1]. It was decided, that the variables adopted in the analysis were as follows: three lengths of compressed wooden bars (x_1, x_2, x_3) and two distances between bars along the radial dimension (x_4, x_5).

Adopting the variables as uniformly distributed, with the boundary values (optimization restriction values) taken accordingly, a number of 1000 calculations of Fuller dome numerical model was performed in SOFiSTiK software [2]. The elements of the geometry of random dimensions (e.g. ordinates of the nodes) were parameterized in such a way that a change in the value of a single variable regenerated the whole geometry automatically (a cylindrical coordinate system was used, due to the axial symmetry of the structure).

Conducting the computational process has proven to be a complicated task, as a strong correlation between the variables was observed (for example, the length of the beams acts strongly on the required prestressing of the adjacent cables). Moreover, the various limitations associated with the structure had to be checked in every step of the standard computation algorithm, thus making the computations more time-consuming.

On the basis of the generated set of samples, an optimized geometry was discerned as the MC methodology sample which generated the lowest value of the result index, defined as the product of dead weight and extreme displacement. The lowest value of the index, equal to 1.2714 kN·mm, was obtained for a structure of a 0.04128 kN dead weight (excluding the timber frame), which gave a maximum displacement of 30.8 mm.

Making use of the SOFiSTiK software RELY module [2], comprising the Strurel COMREL package, the probability of structural failure due to the SLS requirements exceeding was calculated using both SORM and Monte Carlo – based Subset Simulation, in a wide variety of geometrical configurations.

5. Real-life structure verification of the experimental and numerical results

The preliminary nature of this paper is associated with the fact, that the Fuller dome is still to be erected.

The initial Geiger dome served only as an ascertainment, that such a structure will be safe and erectable, moreover it affirmed that the undertaken technological solutions were proper and sufficient.

Finally, an attempt will be made to verify the experimental values and the numerical model results compared with the measurements of an actual design.

A series of experiments and measurements is planned to be conducted on the erected Fuller structure – it is expected that the strains, displacements, eigenvalues and eigenmodes will be surveyed, also the basic load cases will be induced.

The summary and the conclusions of the work are planned to be published in subsequent papers submitted by the Authors.

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