Structural post buckling behaviour with respect to complex geometrical imperfection pattern

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Abstract

In this paper model structures composed of rigid bars where strains are concentrated in connecting elastic joints are proposed. These model structures make possible to derive nonlinear algebraic equations which strictly describe pre- and post-buckling behaviour of structures with various combinations of geometrical imperfections. By the way of numerous examples, influence of imperfections on the structural behaviour is discussed. It was found that for special imperfection patterns the increase in the amplitude of initial geometric imperfection can result in an increase in the value of the critical load defining the bifurcation point. It can also be observed a snap-through phenomenon from an unstable path to the stable one. The observed snap through is caused by transition of the structure from to a lower level of potential energy. In these cases, the initial geometric imperfections can play a positive role resulting in a stable post-buckling behaviour.

Keywords: stability of columns, initial imperfections, eigenvalue problem

1. Introduction

In typical structural elements geometric imperfections can take global and local forms. Global imperfections refer mainly to the bending or twisting of the rod axes. On the other hand, local imperfections may occur in the form of deviations of the dimensions of the individual cross-sectional walls or the shape of the contour of the cross section associated with the local deformation of walls or the symmetric and asymmetric "opening" and "closing" of the cross-section. These imperfections are particularly dangerous when their shape coincides with particular forms of local or distortion buckling mode [2]. Taking into account the actual geometrical imperfections, both global and local, is possible by the finite element method [1], [4]. The general procedure for FEM nonlinear analysis of thin-walled structures using Newton's classical method and the Riks method was formulated by Wriggers and Simo. There are many methods of introducing geometric imperfections into the FEM numerical model. One of the simplest is the equivalent load method in which the force induces in the numerical model the deformation corresponding to the initial geometric imperfections. This method is especially suitable for generating global geometric imperfections, but its disadvantage is the creation of additional stresses, which in fact are not accompanied by the formation of geometric imperfections. Very realistic model of construction imperfections, both global and local, can be obtained by introducing a distorted geometry of the FE mesh by measuring the actual imperfections "in situ". This approach requires the use of highly advanced measuring devices (such as 3D scanners) to create 3D imperfection maps and convert them to FEM programs. Unfortunately, in the case of civil engineering structures, this method is still rarely used because of the relatively high cost. It has one more drawback, namely it allows creation of the initial imperfections pattern only for a given structural element. So it only covers one particular case. Due to the high randomness of imperfections, the results obtained on the basis of deterministically defined imperfections are not always reliable and do not allow to identify potentially dangerous configurations. An alternative method of taking into account imperfections may be the method of shaping the deformed geometry as a linear superposition of the buckling modes obtained from a linear solution of eigenvalue problem [3]. In this approach the amplitude of buckling modes can be scaled by proportional coefficients factors determined arbitrarily based on standard tolerances or on a limited number of measurements of real imperfections.

2. Problem formulation

In order to analyse the complex cases of initial imperfection interactions, a model structure was developed. It consists of four perfectly rigid bars connected by linear elastic hinges with the rotational stiffness k_n, where n=1,2,3 (Fig. 1).

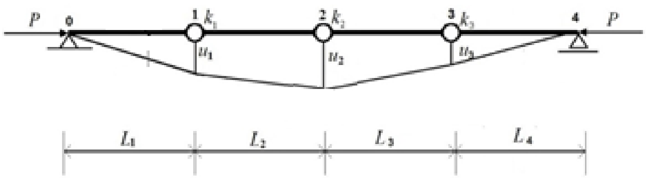

Figure 1: Geometry of model structure

This simple model allows derivation of close-form formulas accounting for large displacements in a form of a set of three nonlinear algebraic equations of equilibrium in post buckling configuration:

$$\begin{cases} k_1\left[as\dfrac{u_1{}^e}{L} - as\dfrac{u_2{}^e - u_1{}^e}{L}\right] = Pu_1 \\ k_2\left[as\dfrac{u_2{}^e - u_1{}^e}{L} - as\dfrac{u_3{}^e - u_2{}^e}{L}\right] = Pu_2 \\ k_3\left[as\dfrac{u_3{}^e - u_2{}^e}{L} - as\dfrac{u_3{}^e}{L}\right] = Pu_3 \end{cases} \tag{1}$$

The above equations represent a non-linear eigenvalue problem in which the displacement of nodes u_n is unknown. The

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terms with superscript *e* represent elastic part of displacements and *as* denotes *arcsin*. The initial geometrical imperfections are introduced into the model in the form of initial displacement u_n^i and do not induce any stress. Thus the total displacements take the form:

$$u_n = u_n^i + u_n^e \tag{2}$$

3. Numerical examples

Numerical examples are solved for various amplitudes of initial imperfections u_n^i (Eq. 3) introduced as a linear combination of eigenvectors u_m^{cr} obtained from the linear eigenvalue problem scaled by mode participation factor α_m.

$$u_n^i = \alpha_m \cdot u_m^{cr} \tag{3}$$

From among many special cases studied by the authors, let us consider the example shown in Fig. 2. This example shows the equilibrium paths obtained for small amplitudes of geometric imperfections in the shape of respectively the first, second and third buckling mode (m1, m2 and m3). It is worth to note that in all three cases the ascending equilibrium paths are observed. Moreover, it is considered that at the bifurcation points the associated unstable equilibrium paths appeared to. When the initial imperfection in the form of first buckling mode is introduced into the model, the bifurcation point correspond almost exactly to the first eigenvalue obtained for the ideal structure. On the other hand, when the imperfections were developed according to the second buckling mode, the bifurcation point correspond to the second eigenvalue , etc. It is therefore apparent that the shape of the initial geometrical imperfection strongly determines the nature of the equilibrium path.

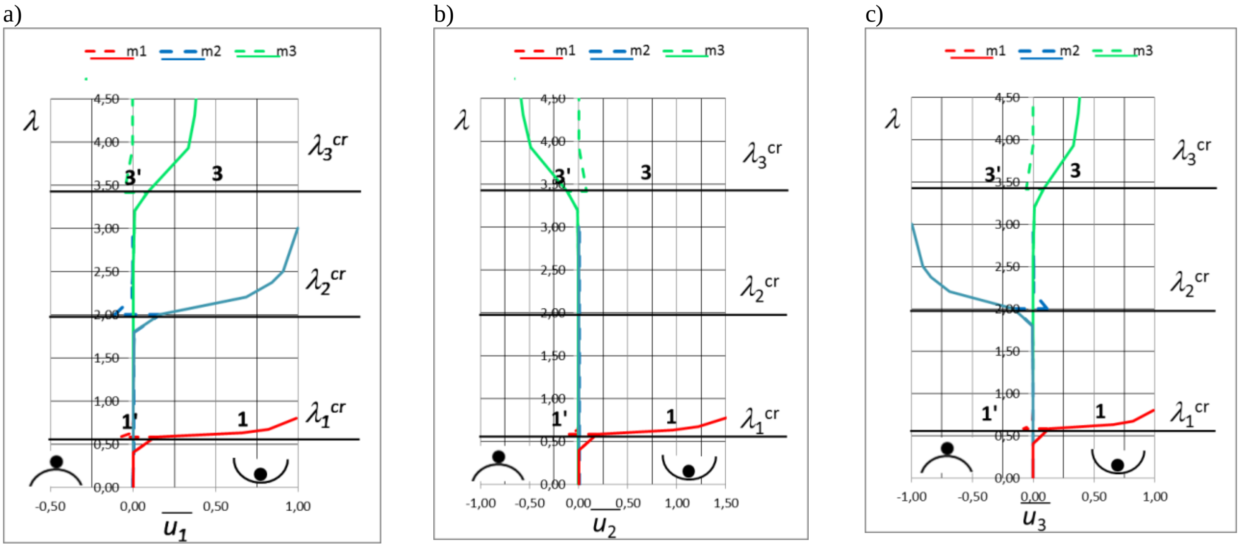

Figure 2: Equilibrium paths obtained for small amplitudes of geometric imperfections in the shape of the first and third buckling modes respectively (m1, m2 and m3); Displacement of the node: a) 1, b) 2, c) 3

4. Concluding remarks

In the paper pre- and post-buckling behaviour of structures with various combinations of geometrical imperfections developed in accordance with three consecutive buckling nodes is discussed. It was found that small amplitudes of geometrical imperfections developed in form of first, second or third buckling mode are accompanied by bifurcation points corresponding to the stable and unstable equilibrium paths. The increase in amplitude of the initial geometrical imperfection results in an increase in the value of the critical load defining the bifurcation point in which it is possible snap through to the configuration described by unstable equilibrium paths. The observed snap through is related to the transition of the structure from a higher to a lower level of potential energy. In this case, the initial geometric imperfections play a positive role in the structural stability, forcing it to remain on a stable equilibrium path. Introduction of the imperfections, developed as a linear superposition of two or three buckling modes, results in the appearance of two or three bifurcation points corresponding to the solution obtained for the perfect structure. Also in this case, the increase in amplitude of the initial geometric imperfections results in an increase in the value of the critical force defining the bifurcation point.

References

[1] Dubina D., Ungureanu V., Effect of imperfections on numerical simulation of instability behaviour of cold-formed steel members, Thin-Walled Structures, 2002, Vol. 40 (3), pp. 239–262.

[2] Rzeszut K., Garstecki A., 2009. "Modeling of initial geometrical imperfections in stability analysis of thin-walled structures", Journal of theoretical and applied mechanics, volume 47,3:667-684.Ainsworth, M. and Oden, J.T., A posteriori error estimation in finite element analysis, *Comput. Methods Appl. Mech. Engng*, 142, pp. 1-88, 1997.

[3] Rzeszut K., Garstecki A., Stability of steel structures with clearances and imperfections. Proceedings of the VII European Congress on Computational Methods in Applied Sciences and Engineering Held in Crete, Greece 5-10 June 2016, ISBN: 978-618-82844-0-1 pp. 4770-4779.

[4] Zeinoddini V., Schafer B.W., Global imperfections and dimensional variations in cold-formed steel members, International Journal of Structural Stability and Dynamics, 2011, Vol. 1 (5), pp. 829–854.