Numerical simulation of FGM materials using functionally graded finite elements

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Abstract

The aim of this paper is numerical modeling of thermoelastic thick cylinder made of FGM, what it means functionally graded composite material, by use of heterogeneous Finite Elements. The main idea of functionally graded materials is a smooth variation of material properties, such as modulus of elasticity, thermal conductivity or thermal expansion coefficient, due to continuous change of microstructure. Heterogeneous layer is usually used as an interface between ceramic coating layer and metal substrate in case when differences of material properties between two constituents are essential. Composite material containing metal as a matrix and ceramic as fibers without FGM interface thin layer may lead to damage or failure due to delamination of the ceramic film from the substrate. This is the result of localized stress gradients at the interface. Numerical methods have to adjust to heterogeneous material. Classical finite element formulation contains constant material properties, which can lead to numerical errors, especially for FGM layer, when the material properties are changing in on huge amount on the thin layer. A proper approach to solve this problem requires application of heterogeneous FE containing additional approximation functions due to interpolate material properties at the level of each finite element. In practice, material shape functions are usually represented by the exponential or power functions, which is approximated the individual character of heterogeneity.

Keywords: heterogeneous, Functionally Graded Material, Finite Element Method

1. Introduction

The material processing have allowed manufacturing special kind of the material composite called functionally graded materials in last few years. The aim of these materials is spatial varying of thermomechanical properties. In this case, FGM are heterogeneous, because the physical properties are changed due to coordinates [1].

Single-component materials are usually characterized either by high thermal resistance like ceramics or high strength, like metals. Composite materials contain two or more constituents (phase), where the ceramic phase is usually characterized by higher thermal and corrosion resistance. Metal or metal alloy phase is characterized by high strength and toughness. However, discontinuity of thermomechanical properties at the coating substrate interface is going to stress concentrations and consequently, transverse cracks or delamination cracks which is caused loss of functionality due to locally removed coating by spalling. As a remedy to this rupture, in 1980 Japanese engineers and scientists involved new generation of ceramic materials, which contains some interface with varying, graded material properties [2].

2. Motivation

Functionally graded composites with smooth variation of volume fractions offer various advantages, such as reduction of residual stress an increase of bonding strength. Moreover, if properly used, such composites also lead to reduction of stress concentration or stress intensity factor. For example, Hasselman and Youngblood [3] found that the maximum tensile thermal stresses in brittle ceramics can be reduced significantly by spatially varying thermal conductivity in hollow circular cylinder subjected to radially inward and outward steady-state heat flow.

The thickness of the FG interface is usually rather small. Moreover, the changes of thermomechanical properties are rather huge. For example, in thermal barrier coating, where the ceramic is usually connected with metallic material, the thermal conductivity for aluminum alloy is equal 154 [W/mK] in comparison to 2 [W/mK] for ceramic material ZrO_2+Y_2O_3, after Wang [4]. This thing consider the functionally graded interface is strong heterogeneous.

Classical finite element formulation contains constant constitutive matrix on the FE level. This assumption leads to errors and numerical simulations of the structure made of functionally graded materials are going to failure. More information and further discussion about classical finite element method in comparison to another numerical method is possible to find in [5].

The aim of this paper is advanced formulation of Finite Element Method connected to heterogeneous material. Author proposes the new type of finite element with double approximation functions. One of them interpolates unknown displacement field and the second one describes changes of heterogeneity of material. The advantages of proposed type of finite element will be shown in the simple numerical example.

3. Graded Finite Element formulation

Kim and Paulino in [6] have proposed special kind of isoparametric finite element to interpolate heterogeneous FGM material. They introduced extra interpolation function to approximate material properties such as Young's modulus $E = E(x)$ and/or Poisson's ratio $\nu = \nu(x)$ as follows:

$$E(x,y) = \sum_{i=1}^{m} N_i(x,y)E_i, \qquad \nu(x,y) = \sum_{i=1}^{m} N_i(x,y)\nu_i \quad (1)$$

where: $N_i(x,y)$ is a material approximate functions similar to shape functions, however its definition should be connected to the type of heterogeneity, m is the number of shape function connected to the type of finite element, E_i and ν_i are the known material data in some interpolation points through the functionally

graded material.

Kim and Paulino have assumed that the interpolation function of material properties is the same as the shape function for isoparametric finite element. On the other hand, the individual character of material composite constituents can be represented by a special kind of function, especially connected to the mixture rule. For example, the appropriate function could be power or exponential functions as follows:

- power function

$$N^m(\xi,\eta) = \xi^{k_1}\eta^{k_2} \tag{2}$$

- exponential function

$$N^m(\xi,\eta) = e^{k_1\xi}e^{k_2\eta} \tag{3}$$

where: k_1 and k_2 are material constants, calculated from the consistency condition on the boundary of FGM interface. Hence, for heterogeneous material the stiffness matrix is given by

$$\mathbf{k}^e = \int_{\Omega^e} (\mathbf{B}^e)^T \mathbf{D}^e(\xi,\eta)\mathbf{B}^e \mathrm{d}\Omega^e \tag{4}$$

where the constitutive matrix contains functions of local coordinates and cannot be shifted in front of the integral.

4. Numerical example of thick FGM cylinder

As a numerical example, the thick, heterogeneous, made of FGM cylinder is consider, where the inner radius $a = 0.1$ [m] and outer radius $b = 0.2$ [m]. It is assumed the plane strain state. The finite model of this cylinder is presented in Fig. 1. The axisymmetry was assumed in this calculations. The cylinder is subjected to internal pressure $p = 80$ [MPa].

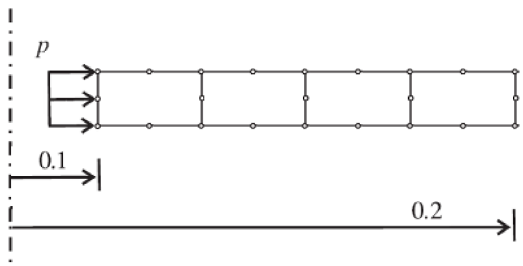

Figure 1: Finite element model of thick cylinder.

The cylinder is made of functionally graded material, where the inner surface is made of aluminum-based composite, called A356R (Al-$Si_7Mg_{0.3}$ + 6% TiB_2) outer surface is made of Cr-Ni steel. The magnitude of Young modulus of A356R is 79 GPa, but Young modulus of Cr-Ni steel is equal 170 GPa. Between these two surface it is the FGM interface. It is assumed exponential function (3) as a material approximation of Young modulus, however the Poisson ratio ν is assumed constant.

Distribution of radial component of stress tensor is presented in Fig. 2. Red line with red square describe the classical solution using homogeneous FE, where the red square stands for Gauss points. On the other hand blue line with diamond corresponds to solution obtained using graded FE, where, similar to previous one, blue diamonds mean the Gauss points. Detailed results of radial stress are presented in Table 1.

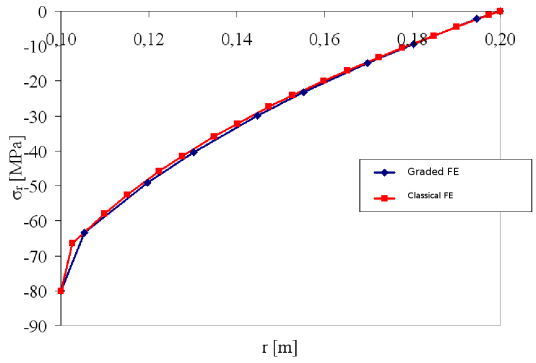

Figure 2: Comparison between classical and graded FE for radial component of stress tensor σ_r

radial coordinate	graded FE	classical FE
0.100000	-80.00000	-80.00000
0.104226	-63.97765	-63.76165
0.115774	-50.64470	-50.00280
0.124226	-42.56945	-41.76345
0.135774	-33.05635	-32.19640
0.143006	-27.71735	-26.96300
0.160327	-17.29965	-16.62805
0.173006	-10.95120	-10.45890
0.190327	-3.482680	-3.317045
0.200000	0.000000	0.000000

Table 1: Comparison between classical and graded FE for the same number of FE.

Deeply analysis of this plot leads to conclusion, that there is necessary to use more finite element in classical way, to obtain similar accuracy as a graded finite elements. Applying the graded FE we can use coarse finite element mesh, hence the time for calculation is less. Magnitudes presents in Table 1 shows that the graded FE are better accuracy to the real heterogeneous material.

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