Numerical and analytical modelling of the end-loaded split (ELS) test for multi-directional coupled laminates

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Abstract

The paper deals with the numerical and analytical modelling of the end-loaded split test for multi-directional laminates affected by the typical elastic couplings. Numerical analysis of three-dimensional finite element models was performed with the Abaqus software exploiting the virtual crack closure technique (VCCT). The results show possible asymmetries in the widthwise deflections of the specimen, as well as in the strain energy release rate (SERR) distributions along the delamination front. Analytical modelling based on a beam-theory approach was also conducted in simpler cases, where only bending-extension coupling is present, but no out-of-plane effects. The analytical results matched the numerical ones, thus demonstrating that the analytical models are feasible for test design and experimental data reduction.

Keywords: multi-directional laminates, elastic coupling, strain energy release rate, end loaded split test

1. Introduction

Multi-directional (MD) laminates made of several layers of variously oriented fibre-reinforced polymer (FRP) laminae, or plies, offer great advantages in many engineering applications from aircraft to marine and automotive industries, as they enable tailoring the structural response to specific design needs. Nonetheless, the widespread use of such structural components is hindered by the requirement of more complex analysis tools with respect to unidirectional (UD) laminates [19], as well as by a poor understanding of the related damage mechanisms and failure modes [4], as testified by the lack of standard testing procedures for the delamination toughness of MD laminates [13].

Modelling difficulties for multi-directional laminates include the presence of elastic couplings between the extension, bending, shear, and torsion deformations and the corresponding internal forces and moments [17,18]. Moreover, like composite structures in general, they are prone to damage phenomena, such as delamination [6,10,14].

In previous studies, theoretical calculations and experiments concerning different delamination fracture modes have been performed for different loads and boundary conditions for laminated beams [11,12]. The main goal was to recognise the influence of a general ply lay-up with different mechanical couplings and boundary conditions on the actual distribution fracture toughness along the delamination front with different ply angles at interfaces. The current work covers analyses of the end loaded split (ELS) test configuration (Fig. 1), defined in the ISO 15114 Standard for unidirectional laminates [7].

Local discrepancies in the stress/strain fields coming from different interfacial fibre-angles generate uneven deformations of the specimens' legs and, consequently, affect the strain energy release rate (SERR) distribution along the delamination front. In general, mixed-mode fracture conditions should be expected [16], so that the adopted fracture criterion should allow for each of the three main fracture modes to be determined and properly considered.

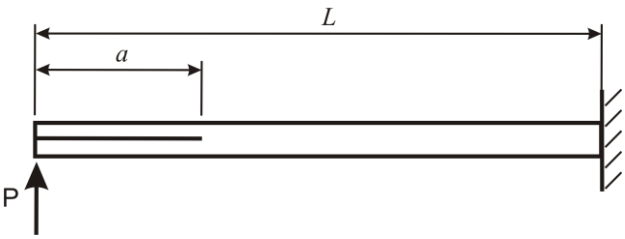

Figure 1. Configuration of the ELS test specimen

2. Mechanical couplings in laminates

According to classical lamination theory (CLT) [8], the internal force, $\mathbf{N}=[N_x, N_{xy}, N_{xy}]^T$, and moment, $\mathbf{M}=[M_x, M_{xy}, M_{xy}]^T$, vectors are related to the strain measures as follows [12]:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \mathbf{A}\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \mathbf{B}\begin{Bmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix}, \quad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \mathbf{B}\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \mathbf{D}\begin{Bmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix}, \tag{1}$$

where ε_x^0, ε_y^0, γ_{xy}^0 and κ_x^0, κ_y^0, κ_{xy}^0 are the strains and curvatures, respectively, of the laminate's mid-plane; **A**, **B**, and **D** respectively are the extensional, coupling, and bending stiffness matrices. By denoting with $\bar{Q}_{ij}^k$ the elements of the elastic moduli matrix of the *k*-th ply (i, j =1...3), with z_k the distances of the ply surfaces from the mid-plane, and with *n* the total number of plies, the elements of the stiffness matrices are:

$$A_{ij} = \sum_{k=1}^{n} \bar{Q}_{ij}^k \left(z_k - z_{k-1} \right) \tag{2}$$

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$$B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \bar{Q}_{ij}^{k}\left(z_k^2 - z_{k-1}^2\right) \tag{3}$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \bar{Q}_{ij}^{k}\left(z_k^3 - z_{k-1}^3\right) \tag{4}$$

Note, that each of the matrices **A** and **D** can only have two forms, reflecting the presence or absence of the in-plane couplings. On the contrary, the out-of-plane coupling stiffness matrix **B** takes one of the six different coupled forms: $\mathbf{B}_L$, $\mathbf{B}_T$, $\mathbf{B}_{LT}$, $\mathbf{B}_S$ and $\mathbf{B}_F$ and the uncoupled one $\mathbf{B}_0$ [18]. This generates a great number of coupled layups and shows the extent of the unexplored field of coupled laminates mechanical properties.

3. Calculations

The calculations of the laminated specimen model were performed both using the finite element method (FEM) and an analytical approach. The stress state and strain energy release rate along the initial delamination front were determined based on linear elastic fracture mechanics (LEFM) [5]. Simulations of the ELS test in the case of coupled laminates enabled distinction of the most problematic couplings from the point of view of a proper calculation of SERR in accordance with the ISO 15114 Standard [7]. Three-dimensional FEM models of ELS test specimens with coupled laminates were analysed with the commercial software Abaqus [1]. Analyses were conducted up to initiation of delamination in accordance with the Benzeggagh-Kennane fracture criterion [3], implemented through the virtual crack closure technique (VCCT) [9,15].

In simpler cases, where only bending-extension coupling is present, but no out-of-plane effects, an analytical approach was also pursued. The energy release rate and its modal contributions were calculated based on both a simplified model, where the sublaminates are rigidly connected, and another one, where an elastic interface is present at the delamination plane [2].

4. Conclusions

The results obtained numerically and analytically for the ELS configuration for multi-directional coupled laminates show dependence of the deflections and SERR distribution on several factors. The most important is the type of coupling, reflected by one of the six forms of the matrix **B**. The intensity of perturbation compared to an uncoupled laminate depends on the values of fibre orientation angles in the stacking sequence. Unexpected occurrence of fracture modes I and III in the ELS test setup designed to provide pure mode II in the case of UD laminated specimens was also observed. These outcomes are in agreement with the results obtained in the ENF study [12]. The analytical results matched the numerical ones, thus demonstrating that a simplified modelling approach is feasible for test design and experimental data reduction.

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