Solid-to-shell transition elements in adaptive analysis of model structures of complex mechanical description

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Abstract

In this work we are interested in effectiveness of application of the solid-to-shell transition elements in adaptive analysis of model structures of complex mechanical description. In this context we assess the model and discretization adaptation of the finite element mesh based on the convergence curves obtained in the three-step *hp*-adaptation process. We check if the assumed global admissible error level is achieved after the final *p*-step of the adaptation. We compare the convergence curves obtained with the use of three types of the transition models. When possible, we also include in this comparison the convergence curves corresponding to the basic models (without transition elements applied). The assessed convergence refers to some chosen model structures problems.

Keywords: solid-shell structure, complex mechanical description, transition models, finite element methods, adaptivity, convergence

1. Introduction

In this paper we assess the effectiveness of the adaptation process in the case of the model problems of simple geometry (plates and shells) and complex mechanical description [3]. This complexity lies in application of different mechanical models in the interior and on the boundaries of the structure. The performed adaptation is based on the three-step adaptive procedure [2,4], controlled with element residual methods [1,4,5]. The applied models are 3D-based ones [3], i.e. only three-dimensional degrees of freedom are applied, regardless of the model type.

The mentioned effectiveness is assessed in the context of the adaptive convergence curves obtained from the models including three versions [3,6,7] of the solid-to-shell transition elements placed between the boundary and internal parts of the model structures. In all three transition models continuity of displacements is guaranteed. The modes differ with the continuity conditions within the strain and stress fields. In these parts the solid (or hierarchical shell) and the first-order shell elements are applied, respectively. To the best of the authors' knowledge such results are not available in the open literature.

2. Numerical research

Here we present the assumptions and results on adaptive convergence of the clamped, uniformly loaded plate problem. The assumptions include: the definition of the approximation error and the reference solution playing the role of the exact one, and setting the adaptivity control parameters. Then, the convergence results of the research are presented and discussed.

2.1. Convergence studies assumptions

It is common to present numerical solution convergence in the form of convergence curves. Such curves are displayed as a graphical representation of the relation between the true approximation error of the solution and the number N of degrees of freedom (dofs) which can be changed through either mesh refinement (h-step of the adaptive procedure) or p-enrichment (p-step of the adaptation). The solution error is defined in a standard way in the strain energy norm, i.e. as a difference of the energies ($U_R - U$). In this difference, U_R denotes the strain energy representing the exact or reference value, while U stands for the strain energy corresponding to the assessed calculation case. When the exact solution value is unknown, the reference value can be taken as the best numerical solution available, i.e. obtained on the finest and the mostly p-enriched mesh.

In the case of the initial mesh we set the uniform longitudinal approximation order ($p = 4$) and the transverse order $q = 1$ or $q = 2$, in the first-order shell domain (and in such parts of the transition domain) and the hierarchical shell domain (and such parts of the transition domain as well), respectively. The reference solution was obtained from the h-adapted mesh with $p = 9$ and $q = 1$ or $q = 2$ in the first-order and hierarchical shell domains, and in the respective parts of the transition domain as well. In the case of the basic or should we say pure models we assume $p = 9$ throughout the entire structure and $q = 1$ in the case of the first-order shell theory or $q = 2$ in the case of the hierarchical shell model.

2.2. Adaptivity control parameters

The adaptation process is performed with the target admissible value $\gamma_T = 0.01$ of the global approximation error. The ratio of the admissible errors from the intermediate h-mesh (γ_I) and the final p-mesh (γ_T) is $\gamma_I/\gamma_T = 2.0$. It is worth noticing that the comparison of the results from the adapted meshes are of qualitative character as the division pattern and the element approximation orders within these meshes look different for each of three variants and the transition model domains cover different elements of these different meshes.

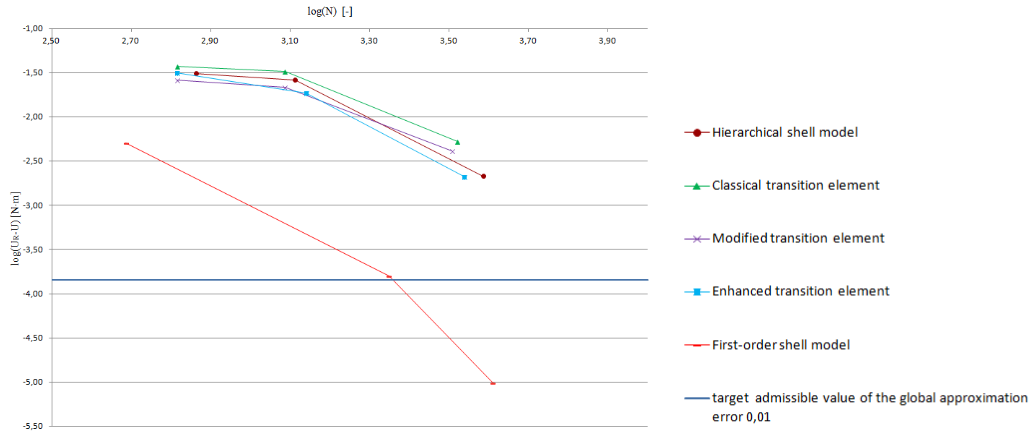

Figure 1: Convergence curves – comparison of the complex and basic models of the plate

2.3. The analysis of the obtained results

The convergence within the adaptation process as a function of the transition element type is presented in Fig. 1. The green curve corresponds to the application of the classical transition element. The violet curve refers to the model with the modified version of the element, while the blue line represents the model with the enhanced transition elements applied.

As it can be seen, the application of the last considered transition element gives the best adaptive convergence of the numerical solution (the lowest error level at the smallest number of dofs). These results are qualitatively consistent with the results obtained from the uniform meshes (uniform *h*- and *p*-convergence studies). Additionally, these three curves can be compared with the curves for the basic models (without transition elements applied). The curve corresponding to the hierarchical shell model is brown, whereas in the case of the first-order shell model it is red. It seems that the hierarchical model curve is qualitatively consistent with the curves of the complex models containing the transition elements. This is due to the fact that in all four cases the hierarchical shell elements are applied on the structure boundaries. Such elements generate a boundary layer along the plate edges. This phenomenon results in lowering the convergence rate with respect to the models where this phenomenon does not occur. A raise of the convergence rate can be obtained only through making the meshes exponentially denser in the direction normal to the boundaries. Note that in the case of the first-order shell, one can observe higher convergence as this model does not generate any significant boundary layer.

3. Conclusions

The assessment of the effectiveness of the adaptation process based on the convergence curves for three types of the transition models applied reveals that the application of the enhanced and modified elements leads to lower error level than in the case of the classical transition element. In the case of the enhanced element one can obtain higher convergence rate (sometimes even much higher) then in the other cases. The convergence rate in the case of the enhanced transition element applied is similar as in the case of the basic model employing the hierarchical shell elements only.

Considering the effectiveness of the adaptation process in the model problems of the plate, one can notice that obtainment of the same global approximation error level from the basic hierarchical shell model and the complex model utilizing the enhanced transition element needs lower number of degrees of freedom in the latter case.

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