On Dedicated Evolutionary Algorithms and Speed-up Techniques Based on Estimation of Convergence Point of Population Applied to Chosen Problems of Mechanics

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Abstract

In this paper we consider several issues regarding our long-term research oriented towards development of highly accelerated and efficient Evolutionary Algorithms (EA) for solving large, non-linear, constrained optimization problems. In particular, we briefly discuss here advances in development of already proposed acceleration techniques, including smoothing and balancing, adaptive step-by-step mesh refinement, as well as a'posteriori error analysis and related techniques. Our recent research has been focused mainly on searching of efficient combination of the proposed techniques and their parameters, as well as on development of some new concepts based on estimation of the convergence point of population [5]. Important engineering applications in mechanics are planned, namely residual stress analysis in railroad rails, and vehicle wheels, as well as a wide class of problems resulting from the Physically Based Approximation of experimental and/or numerical data. The improved EA-based approach provides significant speed-up of solution process and/or possibility of solving such large problems, when the standard EA methods fail.

Keywords: Evolutionary Algorithms, computation efficiency increase, convergence point of population, large non-linear constrained optimization problems, residual stress analysis

1. Introduction

Many important problems of computational mechanics may be formulated in terms of constrained optimization. Complexity of these problems may result mostly from their non-linearity, as well as from a large number of decision variables and constraints involved. Thus, we consider here a wide class of large, non-linear, constrained optimization problems. Due to the size and complexity of such problems, this research is focused, first of all, on the significant efficiency increase of the solution algorithms applied. Our solution approach is based on the Evolutionary Algorithms (EA) [1], which on the contrary to most deterministic methods may be successfully applied to both convex and non-convex problems. However, general efficiency of the standard EA is rather low. Therefore, significant acceleration of the solution process is often needed. We have already proposed and tested several general speed-up techniques with various variants based on simple concepts [3]. Our recent research has been focused mainly on further development of techniques based on estimation of the convergence point of population considered. These techniques were proposed [5] in a general way for almost any population-based algorithms.

The final engineering objective of this long-term research includes residual stresses analysis [2,6] in railroad rails, and vehicle wheels, as well as a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental and/or numerical data [4].

In this paper we also present a brief overview of the proposed speed-up techniques, including advances in their development, and numerical analysis for various demanding benchmark problems. Numerical tests carried out so far indicate significant speed-up of the large optimization processes involved.

2. General problem formulation

In the considered wide class of large, non-linear, constrained optimization problems, a function $u(\boldsymbol{x})$, $\boldsymbol{x} \in R^N$, given in a discrete form, e.g. expressed in terms of its nodal values u_i , $i =1,2,3,\ldots,n$, is sought. These nodal values are defined on a mesh formed by arbitrarily distributed nodes. Here N is the dimension of the physical space of the solution (1D, 2D or 3D), and n is the number of decision variables. The optimal solution usually has to satisfy numerous equality, and inequality constraints. To obtain discrete formulation of optimization problem, any discretization method can be applied, including Finite Element, as well as Meshless Finite Difference Methods used here.

3. EA and acceleration techniques considered

The EA are understood here as real-value coded genetic algorithms consisting of selection, crossover, and mutation operators [1]. Significant speed-up of the EA optimization process can be achieved in various ways, including hardware, software and algorithm improvements. This research is mostly focused now on development of new algorithms, as well as on improvements of certain existing ones. Parallel computations are used as well, but mostly as a support for new speed-up techniques.

We have proposed and tested so far several new, simple but effective EA speed-up techniques, including solution smoothing and balancing, an adaptive step-by-step mesh refinement, as well as a'posteriori solution error analysis and related techniques [3]. Appropriate constraint handling techniques were investigated as well. These general ideas can be applied in various ways. For solution smoothing we have proposed two various approaches - one of them is based on the moving weighted least squares (MWLS) technique, and the second one uses the fitness function enriched by introducing the mean solution curvature [3,4]. Our a'posteriori solution error analysis is based on the stochastic nature of the EA and weighted averaging of the best solutions taken from independent, parallel populations. The improved crossover, mutation, and selection operators take into account information about estimated solution errors.

Our recent research has been focused on further development of techniques based on estimation of the convergence point of population considered. Reference [5] introduces methods for estimation of the convergence point for the moving vectors of individuals between generations. Such convergence point indicates the optimum area. It presents a powerful elite individual for the optimization process [5].

The general idea of estimation of population convergence point is shown in Fig. 1. Considered is a population of M individuals $\mathbf{u}^j = [u_1^j, u_2^j, u_3^j, ..., u_n^j]$, $j = 1,2,3,..., M$ in a n-dimensional space. Moving vectors are calculated between individuals $\mathbf{u}^{j,k}$ from k-th generation and their offspring $\mathbf{u}^{j,k+1}$ from (k+1)-th generation. $\tilde{\mathbf{u}}^k$ is a convergence point, and $\bar{\mathbf{u}}$ is the optimum point.

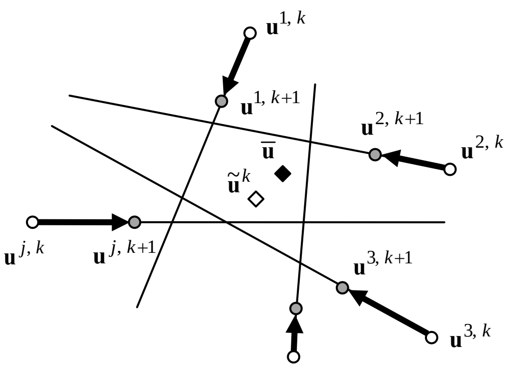

Figure 1: General idea of estimation of the convergence point of population

Various approaches, namely the exact (analytical), approximated (based on the Neumann series expansion), and iterative ones are discussed in [5]. These general methods can be applied to almost any population-based computations. We have proposed and preliminarily evaluated a specific formulation and implementation of these general approaches used for the EA acceleration.

4. Sample of numerical results

The efficiency of new algorithms proposed was examined using various demanding benchmarks involving large number of decision variables and constraints, including residual stress analysis in chosen elastic-perfectly plastic bodies, such as prismatic bar, and thick-walled cylinder, under various cyclic loadings. These benchmarks allow to choose almost any number of decision variables involved. The largest executed numerical tests involved more than 3000 decision variables. Several inverse problems were analyzed as well, including reconstruction of residual stresses. Such analysis used experimentally measured data, and the PBA approach.

In Fig. 2 one may find typical numerical results obtained for our efficiency analysis. The results shown were averaged over 10 independent solution processes. They present convergence of mean solution error for residual stress analysis in the cyclically pressurized thick-walled cylinder used as a benchmark problem.

Techniques based on estimation of the convergence point of population allowed to obtain acceleration up to about 40 times. The best results were obtained for approach using the truncated Neumann series. It is still less than in the case of our earlier approach based on step-by-step mesh refinement combined with smoothing and a'posteriori error analysis (about 140 times) [3], but this methods may be still improved and effectively combined with other ones.

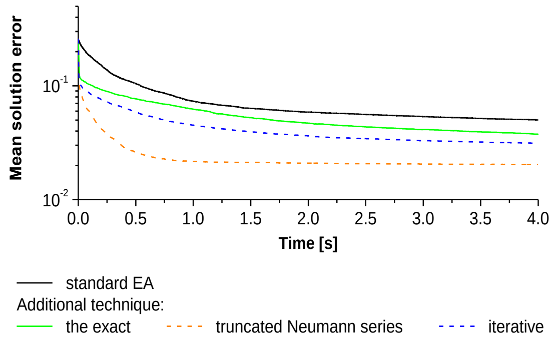

Figure 2: Efficiency analysis of the EA acceleration techniques based on estimation of the convergence point of population

5. Final remarks

Numerical results obtained indicate possibility of practical application of the improved EA to real complex optimization problems involving large number of decision variables and constraints. Numerical analysis also shows possibilities of further development of speed-up techniques considered.

Future research will be mostly focused on application of the improved EA to engineering problems of mechanics, including residual stress analysis in railroad rails and vehicle wheels, as well as the PBA of experimental and/or numerical data smoothing.

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