Optimal repair and reinforcement of plates

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Abstract

This paper is devoted to optimal repair of plates by adding to the existing structure in the current state, some new elements like new elastic or stiff supports, new ribs or other substructures. It is assumed that the primary structure is weakened or damaged and it needs repair and reinforcement. The appropriate optimization problem is formulated as the minimization of increment of the kinematic fields functional induced by introduction of stiffeners, under the cost constraint. In particular, this functional can express the elastic energy or potential energy so the analyzed problem may also represent the maximization of the global structure stiffness. The cost constraint corresponds to condition imposed on the maximal cost of the repair. The method of initial localization of reinforcement is presented and the optimization algorithm is formulated. Numerical examples show usefulness of the applied approach.

Keywords: plates, kinematic fields functional, global stiffness, optimal repair, reinforcement localization

1. Introduction

Often it occurs that the global stiffness or load capacity of a structure in current state is not sufficient. It may be caused by weakening due to operation of some factors like corrosion, mechanical damage, etc. or by changing of working conditions or loading conditions. In such situations it is necessary to repair and/or to reinforce the structure. It can be done by introduction of some new elements like new elastic or stiff supports, new ribs or other additional substructures.

In this paper, at first the problem of optimal repair of plates by their reinforcement is formulated. Next, approach useful for localization of additional elements reinforcing structure is discussed. Also, method of determination of finite increment of the objective functional caused by introduction of finite modification is presented in the case of the elastic energy. Finally, on this basis the algorithm of optimal design of elements reinforcing structure in the current state is proposed and used in numerical example.

2. Formulation of the problem

The problem of optimal repair of the plate in the current, fixed state, in order to minimize increment of arbitrary functional G of kinematic fields, can be presented in the form

$$\min \Delta G \quad \text{subject to} \quad \Delta C \le \Delta C_0 , \tag{1}$$

where the increment ΔG is caused by introduction of additional stiffening elements, which position and parameters will be subjected to optimal design, ΔC is the global additional cost associated with plate reinforcement and ΔC_0 denotes the upper bound imposed on this cost. When

$$\Delta G = \Delta U , \tag{2}$$

where U denotes the elastic energy, problem (1) corresponds to the maximization of the global structure stiffness.

In order to find initial localization of point reinforcement like an elastic support of unknown stiffness k the problem of minimization of topological derivative with respect to support position defined by coordinates x_1, x_2 should be solved, namely

$$\min_{x_1, x_2} \left.\frac{\partial G}{\partial k}\right|_{k=0} . \tag{2}$$

Similarly, in the case of introduction of reinforcement like a rib along unknown line l, the problem of initial localization takes the form

$$\min_{a_i, b_j} \int_l f\left(x_1, x_2, \alpha\right) \mathrm{d}l , \tag{3}$$

where a_i, b_j are the parameters determining respectively location of ends and shape of stiffener, while $f\left(x_1, x_2, \alpha\right)$ is the local topological derivative of the functional G with respect to introduction of rib element at the point $\left(x_1, x_2\right)$ of orientation defined by the angle α (cf. [1]).

Next, when the initial location and initial parameters of reinforcement are determined, any gradient optimization method can be used to find final solution. It is important to notice that usually it only slightly differs from this obtained initially.

In the particular case, when the objective functional corresponds to the elastic energy, the following formula for its finite increment, derived in [2], can be used

$$\Delta U = -\frac{1}{2}[\mathbf{u}_C^0] \cdot \left\langle \mathbf{D}_C \right\rangle^{-1} [\mathbf{u}_C^0] . \tag{4}$$

where $[\mathbf{u}_C^0] = \mathbf{u}_{1C}^0 - \mathbf{u}_{2C}^0$ denotes vector of initial displacements discontinuity at the domain (point) of connection of the plate 1 with stiffener 2, while $\left\langle \mathbf{D}_C \right\rangle = \mathbf{D}_{1C} + \mathbf{D}_{2C}$ is the sum of the local compliance matrices related to the interface, which are submatrices of global compliance matrices $\mathbf{D}_1$, $\mathbf{D}_2$. Let us notice, that during optimization $\mathbf{D}_1$ describing the plate does not change, so only $\mathbf{D}_2$ and $\mathbf{D}_{2C}$ should be separately calculated in each step. It simplifies calculations and the whole iterative process (por. [3]).

3. Optimization algorithm

In order to determine the way of optimal repair of plates by their reinforcement, the algorithm composed of following steps can be applied:
1º Propose general plan of repair of the analyzed plate by choice of the type of reinforcement.
2º Determine initial location of stiffeners using topological derivative (por. [1]).
3º Determine initial parameters of finite stiffening modification under the cost constraint.
4º Using any gradient optimization method, determine optimal values of the design parameters for the reinforcement.
5º Terminate the optimization process or propose another stiffening modification and return to 2º.

4. Numerical example: optimal reinforcement of bending plate by introduction of ribs

Assuming Kirchhoff model, the plate made of steel, shown in Fig. 1a is analyzed. It is fixed on the whole perimeter and loaded by transverse linearly distributed load. The thickness of the plate is 15 mm. The global stiffness of the plate is not sufficient and it should be reinforced. Let us consider the problem of the global stiffness maximization expressed by (1) and (2). Moreover, we assume that reinforcement can by performed by adding one, two or three ribs parallel to the edges and connected to the plate boundaries. The ribs are located symmetrically with respect to the middle surface of the plate.

In the first case, the volume of the ribs is limited to $6 \cdot 10^{-3} m^3$. The ratio of the ribs volume and the initial plate volume is 0.0333. The height of the ribs is constant and equals 100 mm, so only thickness of the ribs change during the optimization. The design parameters are locations of ribs.

The elastic energy of the initial structure is 184.6 J. After introduction of one, two or three ribs the elastic energy decreased to 115.14 J, 82.01 J, 81.71 J, respectively. Optimal ribs positions are shown in Fig. 1b÷1d. In all three cases optimal ribs positions are parallel to the shorter edge of the plate.

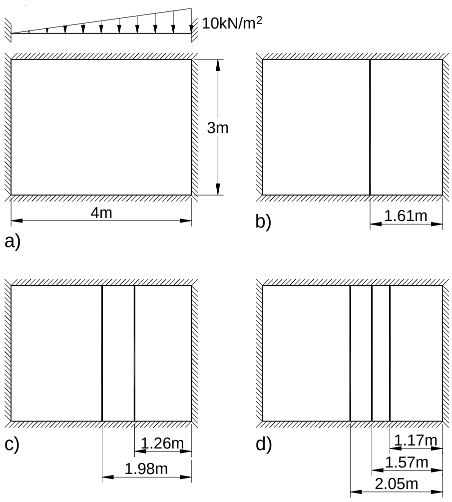

Figure 1: Reinforcement of the plate by ribs of height 100 mm: a) static scheme of the plate, b)÷d) optimal solutions after introduction of one, two and three ribs, respectively

In the second case volume of the ribs is limited to $1{,}8 \cdot 10^{-2} m^3$. Now, the ratio of the ribs volume and the volume of the initial plate is 0.1. Again, height of the ribs is constant. It equals 300 mm and ribs thickness can vary according to their number and length. In this case, when only one rib is introduced, its optimal position is parallel to the longer edge of the plate. The elastic energy equals 30.48 J (Fig. 2a). When rib is located parallel to the shorter edge of the plate its optimal position is shown in Fig. 2b and now, the elastic energy is 38.24 J. But in the cases of introduction of two or three ribs, optimal ribs positions are parallel to the shorter edge of the plate. The optimal ribs positions are shown in fig. 2c÷d. The elastic energy of these structures are 14.68 J and 10.22 J, respectively.

Thus, during transition from introduction of one rib to introduction of two ribs, sudden change of ribs direction occurred.

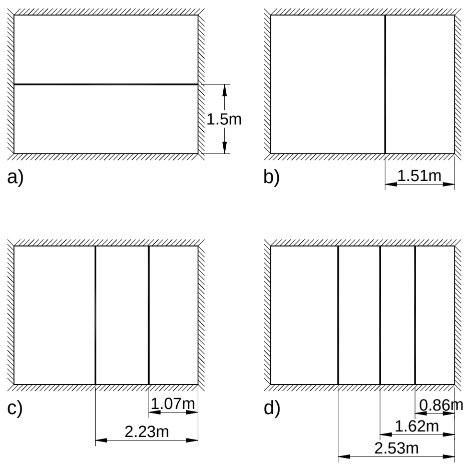

Figure 2: Reinforcement of the plate by ribs of height 300 mm: a)÷b) optimal position of one introduced rib, c) optimal position of two introduced ribs, d) optimal position of three introduced ribs

5. Conclusions

The problem of optimal repair of plates by their reinforcement in order to minimize arbitrary functional of kinematic fields under cost constraint is analyzed in this paper. In particular, this problem may correspond to global stiffness maximization. The algorithm of optimization, with initial localization of modifications, is proposed and successfully used to solve some numerical examples. The presented approach can be also used for another type of finite modification, namely replacement of some weakened or damaged structural elements.

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