Generation of structural topologies using efficient technique based on sorted compliances

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Abstract

The implementation of efficient and versatile methods to the generation of optimal topologies for engineering structural elements is one of the most important issues stimulating progress within the structural topology optimization area. Over years, optimization problems have been typically solved by classical gradient-based mathematical programming algorithms. These traditional techniques are more often replaced by other algorithms, usually based on heuristic rules. Heuristic optimization techniques have gained popularity among researchers because they are easy to implement numerically, do not require gradient information, and one can easily combine this type of algorithm with any finite element structural analysis code. In this paper a novel heuristic algorithm for minimum compliance topology optimization is proposed. Its effectiveness is illustrated by the results of numerical generation of optimal topologies for selected spatial structures.

Keywords: topology optimization, compliance minimization, heuristic algorithm

1. Introductory remarks

Since the early paper by Bendsoe and Kikuchi [1] from late eighties of twentieth century optimization of structural topology is a constantly developing area. Many survey papers and books provide a general overview and a broad discussion on topology optimization concepts, e.g. [5], [6], [7]. One of the most important issues stimulating this progress nowadays is implementation of efficient and versatile methods to generation of optimal topologies for engineering structural elements. Among them there are many heuristic algorithms. Heuristic optimization techniques have gained the great popularity among researchers because they are easy for numerical implementation, do not require gradient information, and one can easily combine this type of algorithm with any finite element structural analysis code. In this paper a novel heuristic algorithm aiming at minimum compliance topology optimization is proposed. The idea of this concept is to implement the local update rule constructed based on values of the specially adapted function of sorted local compliances evaluated for a particular element and for elements forming its neighborhood.

2. Methodology

In topology optimization one searches for a distribution of material within a design domain that is optimal in some sense. In this paper structure compliance:

$$U(\mathbf{d}) = \sum_{i=1}^{N} d_i^p \mathbf{u}_i^T \mathbf{k}_i \mathbf{u}_i \tag{1}$$

is minimized. In (1) $\mathbf{u}_i$ and $\mathbf{k}_i$ are the element displacement vector and stiffness matrix, respectively and N stands for number of elements. The design process consists in redistribution of material and parts that are not necessary from objective point of view are selectively removed. The power law approach defining solid isotropic material with penalization (SIMP) if often adapted with design variables being relative densities of material (see [2]). The elastic modulus E of each element is modelled as a function of relative density d_n using power law: $E_n = d_n^p E_0$, $d_{min} \leq d_n \leq 1$. The power p penalizes intermediate densities and drives design to a material/void structure.

The idea of original heuristic concept proposed in this paper is as follows. Based on results of structural analysis values of local compliances are evaluated for N elements/design elements. Next, compliances are sorted in ascending order and subsets of elements of the smallest and of the largest compliance values are selected. In what follows, N_1, N_2 are specified and $C(n) = -1$ if $n < N_1$ and $C(n) = 1$ if $n > N_2$. For design elements of intermediate compliances $N_{min} \leq n \leq N_{max}$ values of a specially adapted monotonically increasing function are assigned. In this case the linear function of n has been adapted:

$$C(n) = \frac{2}{N_{max} - N_{min}} n - \frac{N_{max} + N_{min}}{N_{max} - N_{min}} \tag{2}$$

The local update rule applied to design element d_n is now constructed based on values of function $C(n)$ evaluated for this element and for M neighboring elements forming user specified neighborhood.

$$d_n^{(t+1)} = d_n^{(t)} + \frac{1}{M+1} \left[\sum_{k=1}^{M} C(k) + C(n) \right] m \tag{3}$$

The quantity m in (3) stands for a move limit.

The numerical algorithm has been built in order to implement the above proposed design rule. As to the optimization procedure the sequential approach, has been adapted, meaning that for each iteration, the structural analysis performed for the optimized element is followed by the local updating process. Simultaneously a global volume constraint can be applied for specified volume fraction $V(\mathbf{d}) = \kappa V_0$, where V_0 stands for design domain volume and κ is the prescribed volume fraction. The volume constraint is implemented in each iteration when local update rules have been applied to all elements. In practice, the design variables multiplier is introduced and then its value is sought for so as to fulfill the volume constraint. As a result, the generated topologies preserve a specified volume fraction of a solid material during optimization process.

3. Results

Selected optimal topologies obtained within the framework of this paper illustrate the proposed concept. As the first example the cuboid structure has been chosen for which regular mesh of $36 \times 50 \times 30$ elements (54000 elements in total) has been applied. Structure is supported in the middle area of a bottom wall as it is shown in Figure 1. Eight concentrated loads of 1 N each are applied, whereas geometrical parameter $a = 1$ m. Material data are as follows: Young modulus 2 10^{11} Pa, Poisson ratio 0.25 and finally, volume fraction $\kappa = 0.4$. Compliance calculated for initial structure which equals: 3.810 10^{-10} Nm serves here as the reference value. Implementation of proposed technique allows to obtain minimal compliance 1.599 10^{-10} Nm for the final topology. For comparison, the same task has been solved using algorithm based on Cellular Automata approach described in [3] and also utilizing internal Ansys topology optimization procedure which is based on optimality criterion approach. In both cases final compliances have larger values than the one obtained within approach of this paper, namely 1.749 10^{-10} Nm and 1.878 10^{-10} Nm, respectively.

Next example is a gear-structure supported on the inner wall of a central hole. The outer ring of thickness 0.0025 m and inner ring of thickness 0.001 m are not subjected to optimization. Two load cases are applied, namely two horizontal concentrated forces acting in opposite directions applied at the lowest and highest point on the circuit and two vertical loads acting in opposite directions applied at the most right and the most left points on the circuit. Each load is equal 1000 N. Structure is discretized with regular mesh of 14400 elements. Material data are as follows: Young modulus 2 10^{11} Pa, Poisson ratio 0.25 whereas the volume fraction $\kappa = 0.5$. A gear structure is 0.008 m thick and geometrical parameter $a = 0.001$ m. Compliance calculated for initial structure which equals 4.289 Nm serves as the reference value. Implementation of proposed technique allows to obtain minimal compliance 2.270 Nm for the final topology. For comparison, the same task has been solved using algorithm described in [3] and utilizing internal Ansys topology optimization procedure. In both cases final compliances have larger values than the one obtained within approach of this paper, namely 2.356 Nm and 2.448 Nm, respectively.

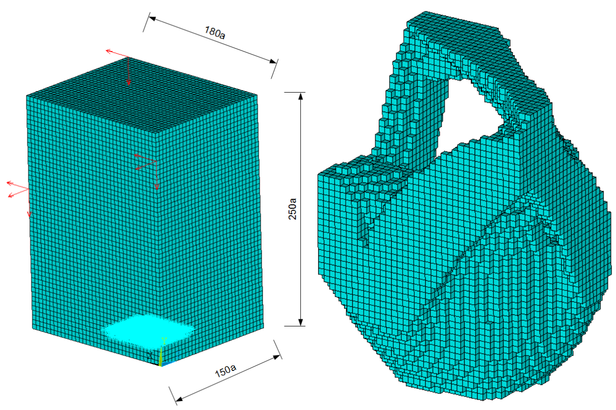

Figure 1: Initial structure (left) and final topology (right)

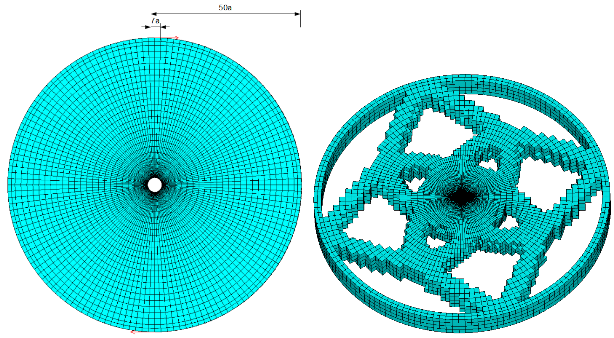

Figure 2: Initial structure (left) and final topology (right)

4. Concluding remarks

Selected examples of generation of spatial topologies of minimal compliance are presented showing effectiveness of the proposed approach. The developed technique is easy to implement, there are only very few parameters to adjust. The algorithm does not require any additional density filtering and generated topologies are free from the checkerboard effect. What has been confirmed, this technique, recently applied to plane structures [4], suits well also generation of spatial topologies. Therefore, it can be concluded that the topology optimization algorithm based on sorted compliances can be considered as a real alternative to other techniques used for generating minimal compliance topologies of engineering structural elements.

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