Determination of stresses in RC eccentrically compressed members using optimization methods

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Abstract

The paper presents analytical and optimization methods for determining the strains and stresses in reinforced concrete (RC) members subjected to the eccentric compression. The governing equations for strains in the rectangular cross-sections are derived by integrating the equilibrium equations of the cross-sections, taking account of the effect of the concrete softening and the mean compressive strength of concrete f_{cm}. The stress-strain relationship for concrete in compression for uniaxial loading is assumed according to Eurocode 2 for nonlinear analysis. This stress-strain relation adequately represents the behavior of the concrete. For reinforcing steel characterized by yield stress f_{yk} linear-elastic model with hardening in plastic range is applied. The task consists in the solving the set of the derived equations s.t. box constraints. The resulting problem was solved by means of fminunc function implemented from the Matlab's Optimization Toolbox. Numerical experiments have shown the existence of many points verifying the equations with a very good accuracy. Therefore, some operations from the global optimization were included: start of fminunc from many points and clusterization. The model is verified on the set of data encountered in the engineering practice.

Keywords: stresses, strains, eccentric compression, reinforced concrete (RC) members, optimization methods, nonlinear equations

1. Introduction

Determination of the normal stresses in the cross-sections of reinforced concrete members subjected to the eccentric compression is considered as a theoretical problem as well as an experimental one. Such members are frequently encountered in the engineering practice (columns, pillars, tower-like structures etc.). Stress determination is a very important stage in the safety assessment of RC existing structures. In order to solve this problem analytically a variety of physical models of materials and methods are applied. For the design of cross-sections the parabola-rectangle diagram for concrete may be assumed. This diagram was used among others by Lechman and Stachurski [1] for determining the normal stresses in the RC annular cross-sections subjected to eccentric compression by means of optimization methods. Despite the generality of the approaches related to the paper topic, there are no appropriate analytical formulae and the solution method for this problem, taking account of the concrete softening, the mean compressive strength of concrete f_{cm} and the secant modulus of elasticity E_{cm}.

2. Equations for rectangular cross-sections

The rectangular RC cross-section is subjected to the axial compressive force N and the bending moment M (Fig. 1). In the derivation of the governing equations the following assumptions are introduced:

- plane cross-sections remain plane
- elasto-plastic stress/strain relationships for concrete and reinforcing steel are used
- the tensile strength of concrete is ignored
- the ultimate strains for concrete are determined as ε_{cu} and for reinforcing steel as ε_{su}.

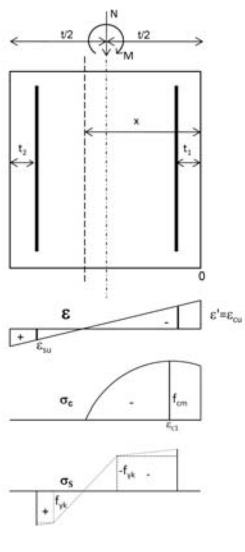

Figure1: Distribution of strain ε, stresses in concrete σ_c and stresses in steel σ_s across the section

In Fig. 1 the following notations are introduced: t, b – the thickness and the width of the cross-section, respectively, t_1, t_2 – coordinates describing the locations of rebars, x, x' – coordinates describing the location of the neutral axis and the location of any point of the section, respectively.
According to Eurocode 2 the stress-strain relation for concrete σ_c - ε_c in compression for uniaxial loading is assumed as:

Table 1: Numerical results

N_{Rm}	M_{Rm}	Concrete cross-section		N_{Rm}	M_{Rm}	RC cross-section	
[kN]	[kNm]	$-5<\varepsilon' \le -10^{-10}$	$10^{-10}< \xi \le 1.0$	[kN]	[kNm]	$-5<\varepsilon' \le -10^{-10}$	$10^{-10}< \xi \le 1.0$
1345.1	76.25	**-3.499275**	**0.699997** ($2\cdot e^{-29}$)	1496.3	118.6	-2.278243	0.772325 ($2\cdot e^{-29}$)
		-1.753891	0.831320 ($4\cdot e^{-29}$)			-3.499766	0.699994 ($7\cdot e^{-13}$)
						-3.499935	**0.699997** ($3\cdot e^{-29}$)

$$\sigma_c = \frac{k\eta_c - \eta_c^2}{1+(k-2)\eta_c} f_{cm} \tag{1}$$

where: $\eta_c = \varepsilon_c/\varepsilon_{c1}$, ε_{c1} – the strain at peak stress on the σ_c - ε_c diagram, k = 1,05 $E_{cm}|\varepsilon_{c1}|/f_{cm}$. This stress-strain relation adequately represents the behavior of the concrete by introducing four parameters: f_{cm} , ε_{c1} , ε_{cu} and E_{cm} . For reinforcing steel, characterized by yield stress f_{yk} , modulus of elasticity E_s, and coefficient of hardening E_h, linear elastic model with hardening in plastic range is applied. In further considerations the corresponding dimensionless coordinates are used: $\xi = x/t$, $\xi' = x'/t$, $\xi_1 = t_1/t$, $\xi_2 = t_2/t$. Let us consider the section under combined compression and bending. The equations for strains in the rectangular cross-sections are obtained by integrating the section equilibrium equations, taking account of the physical and geometrical relationships and considering all possible cases of the stress distribution in concrete and reinforcing steel [2]:

$$\begin{aligned} & n+(1/(k-2))\{W_1\xi+0.5k_2\xi^2-(1/(k-2))[(W_2/W_3)\ln W-\xi]\} \\ & +\mu_1\frac{f_{yk}}{f_{cm}}\left\{\delta_{i1}\left[-1+\frac{E_h}{f_{yk}}((1-\frac{\xi_1}{\xi})\varepsilon'+\varepsilon_{ss})\right]+\delta_{i1+1}\frac{\varepsilon'}{\varepsilon_{ss}}(1-\frac{\xi_1}{\xi})\right\}+ \\ & \mu_2\frac{f_{yk}}{f_{cm}}\left\{\delta_{i2}\left[+1+\frac{E_h}{f_{yk}}((1-\frac{1-\xi_2}{\xi})\varepsilon'-\varepsilon_{ss})\right]+\delta_{i2+1}\frac{\varepsilon'}{\varepsilon_{ss}}(1-\frac{1-\xi_2}{\xi})\right\}=0 \end{aligned} \tag{2}$$

$$\begin{aligned} & -m+(1/(k-2))\left\{\begin{matrix} 0.5(W_1+(1/(k-2)))\xi+0.5[-W_1+0.5k_2-((1/(k-2))]\xi^2+ \\ -(1/3)k_2\xi^3-(W_2/((k-2)W_3))[0.5\ln W+\xi-(W/W_3)\ln W] \end{matrix}\right\} \\ & +\mu_1\frac{f_{yk}}{f_{cm}}(0.5-\xi_1)\left\{\delta_{i1}\left[1+\frac{E_h}{f_{yk}}((1-\frac{\xi_1}{\xi})\varepsilon'+\varepsilon_{ss})\right]+\delta_{i1+1}\frac{\varepsilon'}{\varepsilon_{ss}}(1-\frac{\xi_1}{\xi})\right\}+ \\ & \mu_2\frac{f_{yk}}{f_{cm}}(0.5-\xi_2)\left\{\delta_{i2}\left[1+\frac{E_h}{f_{yk}}((1-\frac{1-\xi_2}{\xi})\varepsilon'-\varepsilon_{ss})\right]+\delta_{i2+1}\frac{\varepsilon'}{\varepsilon_{ss}}(1-\frac{1-\xi_2}{\xi})\right\}=0 \end{aligned} \tag{3}$$

$$n = \frac{N}{bt\, f_{cm}} \qquad m = \frac{M}{bt^2 f_{cm}} \tag{4}$$

$k_2=\varepsilon'/(\varepsilon_{c1}\ \xi)$, $W_1=k-k_2\xi$, $W_2=k(k-2)+1$, $W_3=(k-2)k_2$, $W=1+(k-2)k_2\xi$, $\delta_i=0.5((-1)^i+1)$, $i=1,2$, $\varepsilon_{ss}=f_{yk}/E_s$. (5)

The obtained set of the equations (2)–(3) with the unknown variables ε', ξ describes the strain state in the cross-section subjected to the given axial force N and bending moment M. In a similar way one obtains the equations for the section wholly in compression. The equations (2)–(3) are strongly nonlinear and difficult to be solved analytically, therefore an optimization approach was chosen to obtain an effective solution. Assuming the ultimate compressive strain in the concrete is reached (ε_{cu} =-3.5‰, ε_s =1‰), the formulae (2)-(5) may be used for calculating the values of the ultimate axial force N_{Rm} and bending moment M_{Rm} (b=t=0.3m, ξ_1=ξ_2 =0.1, f_{cm} =28MPa, f_{yk} =500 MPa, E_{cm} =30 GPa, ε_{c1} =-2.0‰, E_h =0, $\mu_1\ f_{yk}/f_{cm}$ =0.1).

3. Solution technique

The task consists in the solving the set of two highly nonlinear equations in the presence of the box constraints on variables that make necessary to apply the least squares technique. Therefore, the functions from the MATLAB Optimization Toolbox were used. Due to appearance of many local minima and several global minima, some operations from the global optimization were included: start of fminunc from many points and clusterization. The selected results are shown in Table 1. Moreover, an additional physical criterion expressed by the Hamilton' Principle has been introduced for choosing the right solution (indicated by the bold font; (·) – the solution accuracy). The calculated compressive stresses in the concrete σ_c versus the normalized eccentricity η = m/n at the ultimate limit state are plotted in Fig. 2, in allowance for the concrete softening. The steel stress in compression was determined as σ_{s1} = 500 MPa, while in tension as σ_{s2} = 0; 200; 500 MPa, respectively.

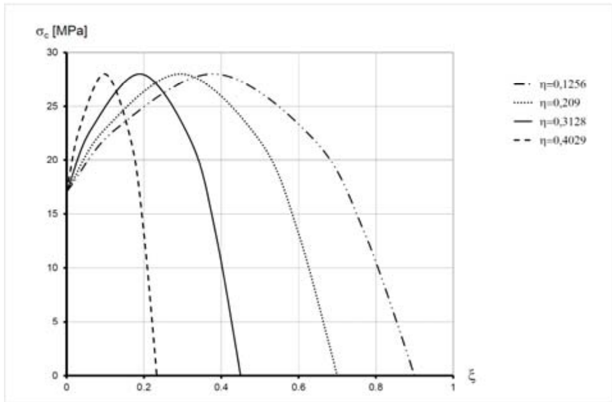

Figure 2: Concrete compressive stresses σ_c in the section under compression and bending versus the normalized eccentricity η

4. Conclusions

The analytical approach combined with the optimization methods have been proposed for determining the stresses in RC cross-sections under eccentric compression. In most analyzed cases 2 - 5 significantly different solutions have been found with a good numerical accuracy. The proposed approach results in more realistic assessment of the strains and stresses in RC cross-sections compared to those based on the parabola-rectangle diagram for concrete [1, 2]. The calculated stresses are significantly influenced by the concrete softening [2]. The presented approach may be applied in the structural design and maintenance of RC structures.

References

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