Aerodynamic calculations of the Sienna Towers buildings complex with respect to human vibrations comfort of their users

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Abstract

The paper presents aerodynamic calculations of the Sienna Towers high buildings complex in Warsaw using authors mathematical model of the considered issue. Human vibrations comfort criteria were checked according to ISO/6897. Dynamic coefficients used in the calculations were obtained from wind tunnel tests.

Keywords: aerodynamic calculations, human vibrations comfort, high buildings vibrations

1. Subject and scope of the paper

The subject of aerodynamic calculations are three high buildings of the "Sienna Tower" complex in Warsaw, Poland (Fig. 1). The main goal of this work is to determine the dynamic response of the buildings caused by turbulent wind action (i.e. buffeting) and then verify user's vibration comfort criteria. The base of these calculations is own mathematical model of the issue.

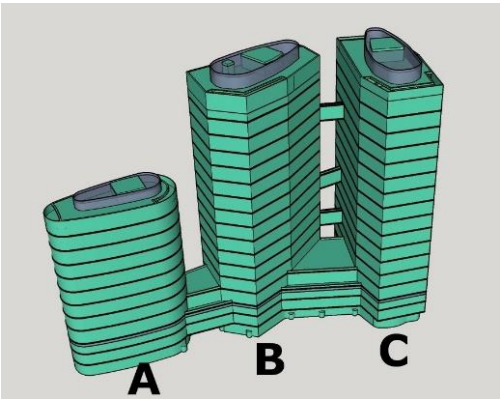

Figure 1: The "Sienna Tower" complex in Warsaw

2. Mathematical model of the problem

In aerodynamic calculations only wind action reduced to components of along- $w_x(z,t)$ and across- $w_y(z,t)$ wind action were taken into account. These components are described by simplified formulas [3]:

$$w_x(z,t) = \frac{1}{2}\rho[\bar{v}(z) + v'(z,t)]^2 D(z)C_x(z) \cong$$
$$\cong \tfrac{1}{2}\rho\bar{v}^2(z)D(z)C_x + \rho\bar{v}(z)v'(z,t)D(z)C_x =$$
$$= \bar{w}_x(z) + w'_x(z,t) \quad (1)$$

$$w_y(z,t) = \frac{1}{2}\rho[\bar{v}(z) + v'(z,t)]^2 D(z)C_y(z) \cong$$
$$\cong \tfrac{1}{2}\rho\bar{v}^2(z)D(z)C_y + \rho\bar{v}(z)v'(z,t)D(z)C_x = \quad (2)$$
$$= \bar{w}_x(z) + w'_x(z,t)$$

$$\bar{v}(z) = \overline{v_H}\Phi_{\bar{v}}(z);\ D(z) = D_H\Phi_D(z) \quad (3)$$

$$a_x = \rho\bar{v}_H D_H C_x; y = \rho\bar{v}_H D_H C_y \quad (4)$$

$$\bar{w}_x(z) = \tfrac{1}{2}\bar{v}_H a_x \Phi_{\bar{v}}^2(z)\Phi_D(z) \quad (5)$$

$$w'_x(z,t) = a_x\Phi_{\bar{v}}(z)\Phi_D(z)v'(z,t) \quad (6)$$

$$\bar{w}_y(z) = \tfrac{1}{2}\bar{v}_H a_y \Phi_{\bar{v}}^2(z)\Phi_D(z) \quad (7)$$

$$w'_y(z,t) = a_y\Phi_{\bar{v}}(z)\Phi_D(z)v'(z,t) \quad (8)$$

$$\frac{\bar{v}(z)}{\bar{v}_H} = \left(\frac{z}{H}\right)^{\alpha}; \bar{v}(z) = \bar{v}_H\left(\frac{z}{10}\right)^{\alpha} = \bar{v}_H\Phi_{\bar{v}}(z); \Phi_{\bar{v}}(z) = \left(\frac{z}{10}\right)^{\alpha} \quad (9)$$

$$\Phi_x(z) = \Phi_H\left(\frac{z}{H}\right)^{\delta_x}, \Phi_y(z) = \Phi_H\left(\frac{z}{H}\right)^{\delta_y} \quad (10)$$

where (comp. Fig. 2): x – mean wind direction; y – horizontal direction; z – vertical direction; ρ – density of the air; $\bar{v}(z)$ – average wind speed; $v'(z,t)$ – wind speed fluctuations; $D(z)$ – characteristic cross-sectional dimension of the structure; C_x, C_y – aerodynamic drag and across-wind action coefficients; $\bar{w}_x(z)$, $\bar{w}_y(z)$ – mean values of along- and across-wind action per unit height of the structure (components of a static wind action); $w'_x(z,t)$, $w'_y(z,t)$ – components of fluctuations (components of a dynamic wind action).

Each of the buildings is treated as a substitute system of one degree of freedom, connected with the first free vibration mode of components $\Phi_x(z)$ and $\Phi_y(z)$.

The maximum values of displacements components are described by [6]:

$$u_x^{max}(z) = \bar{u}_x(z) + g_{ux}\sigma_{ux}(z) =$$
$$= \left[1 + g_{ux}\frac{\sigma_{ux}(z)}{\bar{u}_x(z)}\right]\bar{u}_x(z) = \beta_x\bar{u}_x(z) \quad (11)$$

$$u_y^{max}(z) = \bar{u}_y(z) + g_{uy}\sigma_{uy}(z) =$$
$$= \left[1 + g_{uy}\frac{\sigma_{uy}(z)}{\bar{u}_y(z)}\right]\bar{u}_y(z) = \beta_y\bar{u}_y(z) \quad (12)$$

where: σ_{ux} , σ_{uy} – components of standard deviations of displacements u_x and u_y; g_{ux}, g_{uy} – peak coefficients of wind dynamic action (wind gusts), usually taken from the range of $g_{ux} \in (3,4)$; in average: 3.5; β_x, β_y – coefficients of dynamic along- and across-wind action (called also gust response factors).

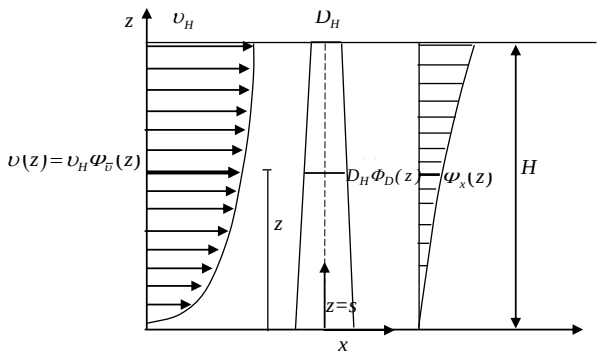

Figure 2: A vertical wind profile $\bar{\upsilon}(z)$, a characteristic cross-section dimension $D(z)$ and x – component of the first flexural vibration mode $\Phi_x(z)$ of the tower-shaped building

Values of gust response factors β_x, β_y were obtained in wind tunnel tests performed for aero-elastic models of the buildings [5].

The maximum values of the displacements components can be also given by:

$$u_x^{max}(z) = \beta_x \bar{u}_x(z) \cong$$

$$\cong \beta_x \frac{q_H D_{śr}}{\omega_1^2 \mu_{śr}} \cdot \frac{\Phi_{Hx}(\Phi_{Hx}C_x + \Phi_{Hy}C_y)}{\Phi_{Hx}^2 + \Phi_{Hy}^2} \cdot \frac{2\delta+1}{2\alpha+\delta+1} \cdot \left(\frac{z}{H}\right)^{\delta_x} \tag{13}$$

$$u_y^{max}(z) = \beta_y \bar{u}_y(z) \cong$$

$$\beta_y \frac{q_H D_{śr}}{\omega_1^2 \mu_{śr}} \cdot \frac{\Phi_{Hy}(\Phi_{Hx}C_x + \Phi_{Hy}C_y)}{\Phi_{Hx}^2 + \Phi_{Hy}^2} \cdot \frac{2\delta+1}{2\alpha+\delta+1} \cdot \left(\frac{z}{H}\right)^{\delta_y} \tag{14}$$

The frequencies and mode shapes of the buildings were obtained as a result of dynamic FEM calculations [5]. Using the results, three modal responses given in accelerations were determined for considered buildings. Next, users vibration comfort criteria were checked accordingly to ISO / 6897 [1,2,4].

A response of high engineering structures under turbulent wind action generally represents a stationary random process in time, of a power spectral density $G_u(f)$, which is dominated by a narrow range of frequencies with extremum for $f = f_1$.

Between power spectral densities of displacement $G_u(f)$, and acceleration $G_a(f)$, of a selected point of the building, the following relation takes place:

$$G_a(f) = (2\pi f)^4 G_u(f) = \omega^4 G_u(f) \tag{15}$$

Taking it into consideration, formulas for standard deviations of the accelerations σ_{ax} and σ_{ay} – that are the basis of the criteria of the vibration comfort – can be estimated as follows:

$$\sigma_{ax}(z, f_x) = \omega_1^2 \frac{\beta_x - 1}{g_{ux}} \cdot \frac{u_x^{max}(z)}{\beta_x} \tag{16}$$

$$\sigma_{ay}(z, f_y) = \omega_1^2 \frac{\beta_y - 1}{g_{ux}} \cdot \frac{u_y^{max}(z)}{\beta_y} \tag{17}$$

3. Aerodynamic calculations and results

Aerodynamic calculations were performed for: three analysed buildings, several worst cases of β coefficient and two states – with and without live load. Exemplary calculations results for the building C, wind angle 340˚ and with live load are put together in Tab. 1.

Table 1: The exemplary results of accelerations calculations for building C at different heights *z*

Building C dead weight and live load			
z[m]	Acceleration *a*[m/s²]		
	X	*Y*	*max*
0.00	0.00000	0.00000	0.00000
18.84	0.00262	-0.00005	0.00262
36.99	0.00630	-0.00013	0.00630
47.88	0.00881	-0.00018	0.00881
58.77	0.01150	-0.00024	0.01151
69.66	0.01435	-0.00030	0.01435
80.55	0.01733	-0.00036	0.01733
91.44	0.02044	-0.00042	0.02044
102.33	0.02366	-0.00049	0.02366
113.22	0.02698	-0.00056	0.02698
124.50	0.03052	-0.00063	0.03053

According to [4], the maximum acceleration for the building C should not exceed a value of 0.044 m/s². As can be seen, a human vibrations comfort criteria are fulfilled.

4. General conclusions

The presented mathematical model can be used in the analysis of vibration comfort of tower-shaped buildings users.

5. References

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