Soft tissue freezing process. Identification of the dual-phase lag model parameters using the evolutionary algorithm

Bohdan Mochnacki[1] **Ewa Majchrzak**[2] **and Marek Paruch**[2]

[1] *University of Occupational Safety Management in Katowice*
Bankowa 8, 40-007 Katowice, Poland
e-mail:bmochnacki@wszop.edu.pl

[2] *Silesian University of Technology*
Konarskiego 18A, 44-100 Gliwice, Poland
e-mail:ewa.majchrzak@polsl.pl, marek.paruch@posl.pl

Abstract

In the paper the soft tissue freezing process is considered. The tissue sub-domain is subjected to the action of cylindrical cryoprobe. Thermal processes proceeding in the domain considered are described using the dual-phase lag equation (DPLE) supplemented by the appropriate boundary and initial conditions. DPLE results from the generalization of the Fourier law in which two lag times are introduced (relaxation and thermalization times). The aim of research is the identification of these parameters on the basis of measured cooling curves at the set of points selected in the tissue domain. To solve the problem the evolutionary alghoritms are used. The paper contains the mathematical model of the tissue freezing process, the very short information concerning the numerical solution of the basic problem, the description of the inverse problem solution and the results of computations

Keywords: tissue freezing, dual-phase lag model, inverse problem, evolutionary algorithms, numerical methods

1. Introduction

The typical tissue freezing model is based on the Pennes equation (e.g. [1, 2]) in which the parameter called a substitute thermal capacity is introduced [3, 4]. Recently, however, prevails the view that a better approximation of the processes proceeding in the domain of the heated or cooled soft tissue are the models using the Cattaneo-Vernotte equation [5] or the dual- phase lag equation [4, 6]. It results from the specific inner structure of the material considered.

The DPLE contains the second derivative of temperature with respect to time and also the mixed derivative both in time and space**.** The typical boundary conditions given on the outer surface of the system have also the another form than the classical ones. As mentioned, the equation discussed contains two additional parameters this means lag times τ_q and τ_T (relaxation and thermalization times).

To take into account the tissue freezing process the approach called the one domain method (e.g. [3, 4]) can be applied. The evolution of the freezing latent heat is determined by the substitute thermal capacity. It turned out that the same parameter can be introduced to the dual-phase lag equation (see: next Chapter).

At the stage of numerical modeling of the basic problem the finite difference method has been used. In particular, the 1D problem for the domain oriented in the cylindrical co-ordinate system has been considered (it results from the experimental data available in the literature [7] and concerning the tissue freezing exposed to the cylindrical cryoprobe action).

The inverse problem consisting in the determining of lag times was solved using the evolutionary algorithms (e.g. [8]).

In the final part the results of computations are shown and the conclusions are formulated.

2. Governing equations

The dual-phase lag equation results from the generalized form of the Fourier law $\mathbf{q}(x, t) = -\lambda \nabla T(x, t)$, this means

$$\mathbf{q}(x, t+\tau_q) = -\lambda \nabla T(x, t+\tau_T) \tag{1}$$

where $\mathbf{q}$ is a heat flux vector, λ is a thermal conductivity, x, t denote the geometrical co-oridinates and time.

After not very complex mathematical manipulations one obtains the diffusion equation in the form

$$c\left[\frac{\partial T(x, t)}{\partial t} + \tau_q \frac{\partial^2 T(x, t)}{\partial t^2}\right] = \nabla\left[\lambda \nabla T(x, t)\right] +$$

$$\tau_T \nabla\left[\lambda \frac{\partial \nabla T(x, t)}{\partial t}\right] + Q(x,t) + \tau_q \frac{\partial Q(x, t)}{\partial t} \tag{2}$$

where c is a volumetric specific heat, $Q(x, t)$ is a capacity of internal heat sources, in particular

$$Q(x,t) = w_B(T) c_B [T_B - T(x,t)] + Q_m(T) + L \frac{\partial f_S(x,t)}{\partial t} \tag{3}$$

where $w_B(T)$ [kg/(m^3 s)] is the blood perfusion rate, c_B is the specific heat of blood, T_B is the arterial blood temperature, $Q_m(T)$ is the metabolic heat source, L is a freezing latent heat, f_S is a frozen state fraction at the neighborhood of the point considered. A form of perfusion heat source results from the assumption that the soft tissue is supplied by the big number of capillary blood vessels uniformly distributed in the tissue domain. The local capacity of heat source connected with the freezing process is proportional to the local freezing rate [4]. It should be pointed out that the blood perfusion rate and the metabolic heat source are equal to zero for the frozen region ($T < T_2$), while for the intermediate one the linear changes starting from the point

The paper is a part of Research Project 2015/19/B/ST8/01101 sponsored by National Science Centre (Poland).

$T = T_1$ (corresponding to the natural state of tissue) are taken into account.

The internal heat source connected with the freezing process can be transformed to the form

$$Q_F(x,t) = L\frac{\partial f_S(x,t)}{\partial t} = L\frac{\mathrm{d}\, f_S(T)}{\mathrm{d}T}\frac{\partial T(x,t)}{\partial t} \tag{4}$$

If the course of the function $f_S(x, t)$ in the intermediate region is assumed to be a linear one

$$f_S(x,t) = \frac{T_1 - T(x,t)}{T_1 - T_2} \tag{5}$$

then in the equation (2) in the place of c the parameter

$$C = c_P - L\frac{\mathrm{d}\, f_S(T)}{\mathrm{d}T} = c_P + \frac{L}{T_1 - T_2}, \qquad T \in \left[T_2,\, T_1\right] \tag{6}$$

called a substitute thermal capacity appears. Additionally the last term in equation (3) and its derivative $\partial Q_F(x, t_q)/\partial t$ disapears. In equation (6) c_P is the volumetric specific heat of intermediate sub-region. Finally, in equation (3) the parameter C has a form of the piece-wise constant function. The similar course of the thermal conductivity is also assumed.

On the contact surface between cryoprobe tip and skin tissue the Dirichlet condition (Fig. 1) is given, while on the external part of the cylindrical domain the adiabatic boundary condition is taken into account. Additionally, for $t = 0$ the initial tissue temperature and the initial cooling rate are known.

At the stage of numerical computations the authorial program basing on the explicit scheme of the FDM for axially-symmetrical domains has been used.

3. Inverse problem

To solve the inverse problem the least squares criterion is applied

$$S\left(\tau_q, \tau_T\right) = \frac{1}{M}\sum_{i=1}^{M}\sum_{f=1}^{F}\left(T_i^f - T_{di}^f\right)^2 \tag{7}$$

where T_{di}^f and $T_i^f = T\left(x_i, t^f\right)$ are the known temperatures distribution and estimated temperatures, respectively, M is the number of sensors. The minimum of functional (7) has been found using the evolutionary algorithms (e.g. [8]). In this paper the problem of τ_q and τ_T identification is discussed (c.f. eq. (2)).

In Table 1 the evolutionary algorithm parameters are collected. In Figure 1 the cooling curves at the points located at different distances from the cryoprobe surface and experimental data are shown, while in Figure 2 the process of identification using the evolutionary algorithm is presented.

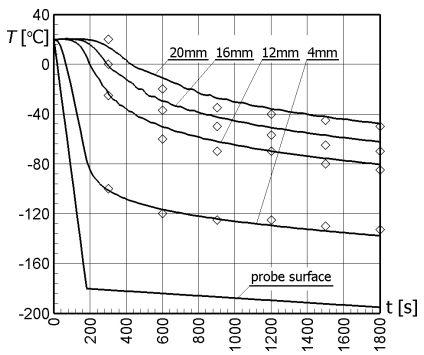

Figure 1: The cooling curves and experimental data

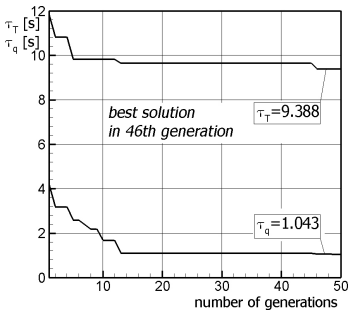

Figure 2: Inverse problem solution using evolutionary algorithm

Thermophysical parameters of tissue are taken from [4]. The radius of croprobe tip equals 4 mm. The cryoprobe is surrounded by the tissue with the radius 0.1 m (this large size is intended to provide the correctness of assumption concerning the adiabatic conditions on the lateral surface of the domain).

Table 1: Evolutionary algorithm parameters

No of generations	30
Number of chromosomes	10
Prob. of uniform mutation	20%
Prob. of nonuniform mutation	30%
Prob. of arithmetic crossover	50%
Prob. of cloning	10%

4. Conclusions

Generally speaking the solution of the inverse problem discussed on the basis of the experimental data concerning the local and temporary temperatures in the tissue domain is quite satisfactory.

References

[1] Pennes, H.H., Analysis of tissue and arterial blood temperatures in the resting human forearm, *J. Appl. Physiol.* l, pp. 93-122, 1948.

[2] Majchrzak, E., Mochnacki, B., Jasiński, M.,Numerical modeling of bioheat transfer in multi-layer skin tissue domain subjected to a flash fire, *Comp. Fluid Solid Mech.*, 1-2, pp. 1766-1770, 2003.

[3] Dombrovsky, L.A., Nenarokomova, N.B., Tsiganov, D.I., Zeigarnik, Y.A., Modeling of repeating freezing of biological tissues and analysis of possible microwave monitoring of local regions of thawing, *Int. J. Heat Mass Transf.*, 89, pp. 894–902, 2015.

[4] Mochnacki, B., Majchrzak, E., *Numerical modeling of biological tissue freezing process using the Dual-Phase-Lag Equation*, in: Advances in Mechanics (M. Kleiber et al. – eds), CRC Press/Balkema, pp. 413-417, 2016.

[5] Cattaneo, M.C., A form of heat conduction equation which eliminates the paradox of instantaneous propagation, *C.R. Acad. Sci. I – Math.*, 247, pp. 431-433, 1958.

[6] Özişik, M.N., Tzou, D.Y, On the wave theory in heat conduction, *J. Heat Transf. Trans. ASME*, 116, pp. 526-535, 1994.

[7] Baust, J., Gage, A.A., Ma, H., Zhang, Ch. M., Minimally Invasive Cryosurgery-Technological Advances, *Cryobiol.*, 34, pp. 373-384, 1997.

[8] Mochnacki, B., Paruch, M., Estimation of relaxation and thermalization times in microscale heat transfer, *JTAM*, 51, 4, pp. 837-845, 2013.