# Transformation and optimization of quadrilateral lattice shells

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Abstract

Free-form design (FFD) became recently a new paradigm of architectural style. Freedom in the design results in a situation when created forms are not always rational from the viewpoint of design logic. One of the basic issues, especially in the case of glazed grids, is the ability to divide the designed surface into a grid composed of planar panels. Most frequently, FFD implementations are forms based on triangulated grids. This type of topology is easy to achieve, since any three points lying on any surface also lie on the same plane. In contrast, obtaining quadrilateral topologies which preserve planarity of panels is much more difficult, and their base surfaces must meet a number of constraints. However, the quadrilateral topology has many advantages over the triangular one, both for implementation and aesthetic reasons. In this paper the method of forming quadrilateral planar grids on surfaces with two-way curvature is proposed and discussed. The advantages of this method are the ability to transform a grid to adapt it to the requirements of architectural form and its optimization due to deformation.

*Keywords: free-form design, doubly curved lattice shells, planar quadrilateral lattice shells, glass canopy*

## 1. Comparison of triangular and quadrilateral topologies

The glazed planar grids based on doubly curved surfaces differ in many respects depending on whether their topology is triangular or quadrangular. Triangular grids are much easier to design and virtually every surface can be triangulated. This fact, rather than any advantages in performance, is the reason why structures with such a topology are much more often designed and constructed. For any free-formed surface, arbitrarily selected three points lying on it also lie on the same plane. This means that any arbitrarily selected three points creates a flat panel. To maintain planarity of panels defined by four points lying on the surface of the shell, these points have to meet additional conditions which, to some extent limit the freedom of shaping the base surface. The use of quadrilateral grids, however, has a strong practical justification. In the case of glazed lattice shells quadrilateral panels significantly facilitate construction. This is due to the reduction of glass losses due to the cutting of the panels and easier workmanship. In the triangular grids nodes usually connect 6 rods, while in quadrilateral grids nodes are 4-valent. For this reason, the latter connections are much simpler in design and fabrication. As a result, the average number of rods in the quadrilateral grids is about fifty percent smaller than in triangular grids. This facilitates the proper fabrication of edge profiles, and thus reduction of thermal bridges and places of potential leaks [3].

## 2. Construction of constrained quadrilateral lattice shells

### *2.1. Construction of the profile curve*

Because of the need to ensure coplanarity of quadrilateral panel vertices, the procedure of shaping the base surface is subject to certain constrains. One possibility to meet these limitations is to use a translational surface created by translating the generatrix curve along the directrix curve [1]. In the case where the structure contour is rectangular, the profiles of both curves must be of the same shape and be constructed in accordance with the rules given below. In Fig. 3, the bold diagonal contour marks the intersection of the generated surface and the base plane. Part of the surface outside the contour is discarded. The profile curves must consist of four identical parts. The half of the profile curve is shown in Fig. 1. The basic part of the profile curve, $C_1$, which is its quarter, is not subject to constrains. Here, for example, the shape parameter is the location of the control point $P_e$. By manipulating it, the shape of the final lattice shell can be changed, Fig. 2. The second quarter of the profile, $C_2$, is formed by rotating the $C_1$ curve around the center at $x = L/2$ and $y = H/2$, by the angle 180°. The remaining part of the profile is constructed by the mirror symmetry with respect to the axis $X = L$.

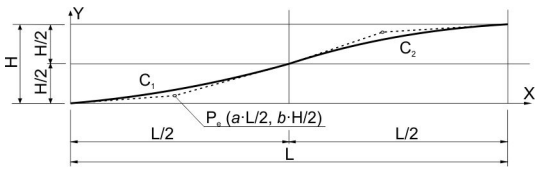


Figure 1: Half of the profile curve

Figure 2: Exemplary profile curve variations

### *2.2. Lattice network generation*

The selection of vertices on the base surface is carried out in such a way that the vertices defining each panel lie both on this surface and on a certain common plane. Coordinates of vertices

result from the intersection of two profile curves: $v = v(G,D)$ (generatrix $G$, and directrix $D$) then:

$$v_1 = v(G_1, D_1) \qquad v_3 = v(G_2, D_2)$$
$$v_2 = v(G_1, D_2) \qquad v_4 = v(G_2, D_1)$$

Vertices $v_1$, $v_2$, $v_3$, $v_4$ create a planar face (Fig. 3).

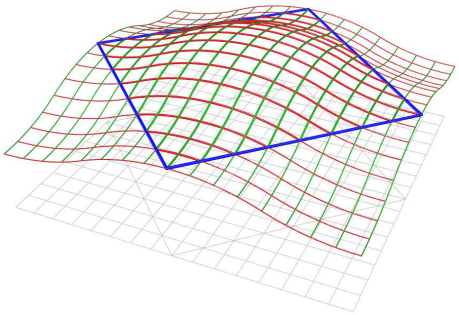

Figure 3: Network of profile curves generating planar panels, bold diagonal outline is the base plane of the lattice shell

### *2.3. Transformations of lattice shell*

A grid with planar panels and square base was generated by the procedures described above. It is the basic form for generating lattice shells based on other shapes. For this purpose, it is necessary to use appropriate transformations which adapt the originally generated grid to the altered base shape while maintaining planarity of grid panels. Any linear transformations such as translation, rotation, scaling in any direction and combinations thereof are allowed. Scaling along the $Z$ axis determines the height of the lattice shell and is also an optimization parameter for the structure. The grid can be inscribed into any quadrilateral base contour, by means of nonlinear transformations, the three examples of which are shown in Fig. 4. The final allowed transformation is "bending", which allows the two sides of the quadrilateral base to be converted into curves, Fig. 5.

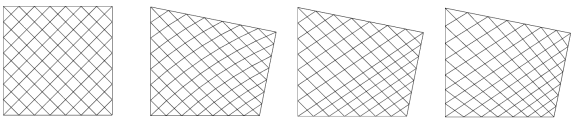

Figure 4: Three types of non-linear transformations preserving planarity of faces

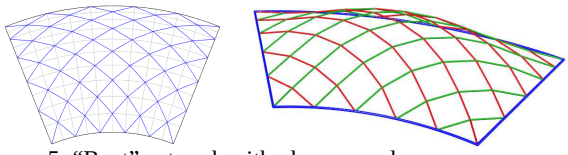

Figure 5: "Bent" network with planar panels

### *2.4. Composed lattice shells on irregular layouts*

Further possibilities of grid shaping are due to the possibility of combining several transformed lattice shells by welding them in such a way that the continuity $G_0$ is maintained. An example is shown in Fig. 6.

Figur 6: Lattice shell composed of three bent parts welded continuously

## 3. Initial rating of mechanical performance of various lattice shell grids

Initial structural efficiency evaluation of the generated lattice shell was carried out by means of FEM software Karamba [2]. The computational model assumed plane dimensions $A \times B$ = 14.14×14.14 m, the length of each rod in plane – 1.00 m and its cross section – aluminium RHS 100×50×4 mm. The self-weight of the grid was assumed as the loading. The analyses compared, for different combinations of parameters, the values of: average and max nodal displacement, max axial forces, max bending moments and max shear forces in the lattice shell. Table 1 presents the maximum grid node displacement values for $H = 2.50$ m and for various combinations of profile curve parameters $a$ and $b$. These values are selected from the full set of results as the most characteristic.

Table 1: Max displacements of nodes [mm] for $H$ = 2.50 m

| $b$ \ $a$ | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 |
|---|---|---|---|---|---|---|---|---|
| 0.00 | 1.71 | 1.08 | 1.28 | 1.58 | 2.01 | 2.60 | 3.40 | 4.41 |
| 0.05 | 2.86 | 2.02 | 1.31 | 1.12 | 1.36 | 1.71 | 2.21 | 2.90 |
| 0.10 | 4.50 | 3.43 | 2.47 | 1.66 | 1.14 | 1.24 | 1.49 | 1.89 |
| 0.15 | 6.81 | 5.42 | 4.20 | 3.10 | 2.15 | 1.42 | 1.33 | 1.45 |
| 0.20 | 9.99 | 8.30 | 6.69 | 5.21 | 3.94 | 2.83 | 1.96 | 1.57 |
| 0.25 | 14.03 | 12.05 | 10.13 | 8.30 | 6.58 | 5.05 | 3.73 | 2.68 |

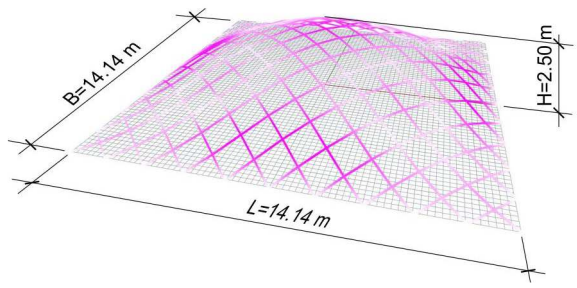


Figure 7: Graph of grid displacements for the case presented in Table 1. The more vivid rod colour means greater displacement

## 4. Concluding remarks

The presented planar quadrilateral lattice shell shaping method allows the deformation and optimization of architectural objects of various use, while maintaining the rationality of designed free-forms.

## References

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