Effect of sub-loops hysteresis in SMA ear system

Andrzej Mitura[1]* **and Rafal Rusinek**[2]*

[1] *Mechanical Engineering Faculty, Lublin University of Technology*
Nadbystrzycka 36, 20-618 Lublin, Poland
e-mail: a.mitura@pollub.pl

[2] *Mechanical Engineering Faculty, Lublin University of Technology*
Nadbystrzycka 36, 20-618 Lublin, Poland
e-mail: r.rusinek@pollub.pl

Abstract

In this paper a numerical model of shape memory alloys with pseudoelastic effect is presented. A new numerical procedure is used to model an external and internal hysteresis loops. The model bases on conditions *if(...)*, which allows to obtain piecewise linear curve. Finally, the shape memory element will be included in the model of the damaged human middle ear to reconstruct ossicular chain.

Keywords: Shape memory alloys, pseudoelastic effect, numerical model, middle ear

1. Introduction

SMAs are materials, which exhibit a reversible thermo-elastic transformation between martensite and austenite influenced by the stress or temperature factor. When the alloys change their shape under the influence of temperature the phenomenon is called as shape memory effect (SME). SMEs can be divided into two types: one- or two-way.

There is also an additional mechanism in SMA, the so called pseudoelasticity [1, 2]. The pseudoelastic effect is presented as a nonlinear dependence of strain and stress in the form of hysteresis loop, which can have internal sub-loops [3,4].

Generally, the thermo-mechanical properties of SMAs can be modelled at different scales. In microscopic and mesoscopic approaches the material behaviour is modelled starting from the molecular and lattice levels, respectively. Other class of models is based on a macroscopic approach, where only phenomenological features of the SMAs are taken into account [5]. Most often SMA models ignore internal sub-hysteresis, which can be important in practice. Therefore, in this paper a new description of this effect is proposed. A mathematical model of pseudoelasticity was developed using conditional functions. Finally, the SMA model with pseudoelasticity is used in the numerical research to obtain dynamic responses of middle ear with SMA prosthesis.

2. Model of the SMA spring

The experimentally obtained characteristic of SMA demonstrates pseudoelastic effect which is modelled here the linear piecewise curve (Figure 1). Figure 1 shows an idealized curve of the external hysteresis loop and adopted characteristic points. Basing on coordinates of points (σ_1,ε_1), (σ_2,ε_2), (σ_3,ε_3), (σ_4,ε_4) the coefficients of lines slopes can be calculated:

$$a_1 = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} \qquad b_1 = \sigma_1 - a_1\varepsilon_1 \tag{1}$$

$$a_2 = \frac{\sigma_3 - \sigma_4}{\varepsilon_3 - \varepsilon_4} \qquad b_2 = \sigma_3 - a_2\varepsilon_3 \tag{2}$$

$$a_0 = \frac{\sigma_4 - \sigma_2}{\varepsilon_4 - \varepsilon_2} \qquad b_0 = \sigma_2 - a_0\varepsilon_2 \tag{3}$$

These coefficients are used to develop the new model with sub-loops which are important during loading and unloading of the SMA between σ_{min} and σ_{max} (Figure 2b and 3b). The diagonal line ($\sigma=b_0+a_0\varepsilon$) is applied to determine turning points of internal sub-loops.

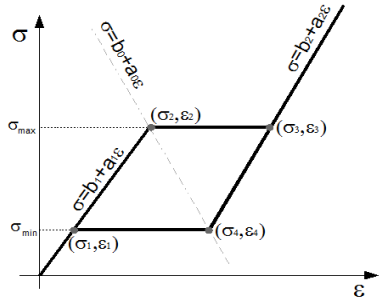

Figure 1: External hysteresis loop $\sigma=\sigma(\varepsilon)$

Generally, new model is described by the equation

$$\sigma = b + a\varepsilon \tag{4}$$

where $\sigma=\sigma(\varepsilon)$, $b=b(\varepsilon)$ and $a=a(\varepsilon)$. In the proposed model the parameters *a* and *b* are checked in each simulation step. During calculation their values can be reinitialized. The change of these values depends on the defined conditions. The stages of numerical procedure used in the description of external and internal hysteresis loops are as follows.

- At time t=0s initial values of *a*, *b* and σ are found.
- At next time steps the values of *a* and *b* are reinitialized according to the conditions:

1) if ($\dot{\varepsilon} \geq 0$ and $\sigma \leq a_0\varepsilon + b_0$) then $a = a_1$, $b = \sigma - a_1\varepsilon$

2) if ($\dot{\varepsilon} < 0$ and $\sigma \leq a_0\varepsilon + b_0$) then $a = 0$, $b = \sigma$

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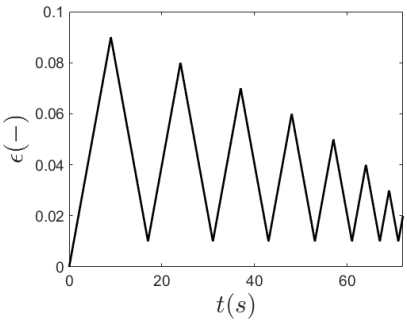
3) if ($\dot{\varepsilon} \geq 0$ and $\sigma > a_0\varepsilon + b_0$) then $a = 0$, $b = \sigma$

4) if ($\dot{\varepsilon} < 0$ and $\sigma > a_0\varepsilon + b_0$) then $a = a_2$, $b = \sigma - a_2\varepsilon$

5) if ($\sigma \geq \sigma_{\min}$ and $\sigma \leq a_2\varepsilon + b_2$) then $a = a_2$, $b = b_2$

6) if ($\sigma \leq \sigma_{\max}$ and $\sigma \geq a_1\varepsilon + b_1$) then $a = a_1$, $b = b_1$

a)



b)

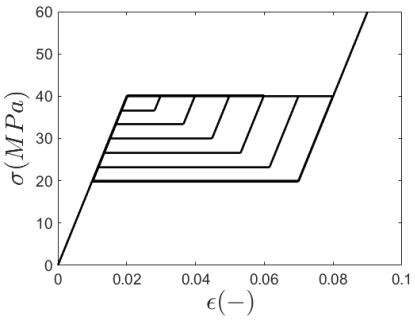


Figure 2: Internal hysteresis loops (first case): a) time series of strain, b) stress versus strain.

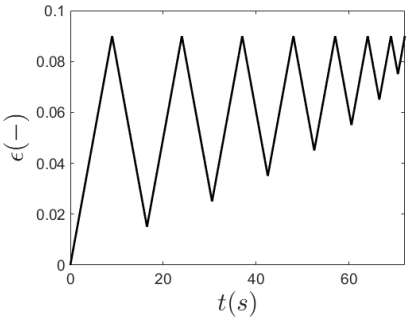
Using the above scheme to describe the hysteresis phenomenon, a numerical model was developed in the Matlab/Simulink software. Figure 2 and 3 present strain (ε) and stress (σ) for two loading paths (case 1 and 2).

The two different paths are selected to show the effectiveness of internal hysteresis loops modelling. Next, the presented model of SMA will be used in the human middle ear as a prosthesis which connects damaged ossicles.

3. Conclusions

The shape memory alloys are materials with controlled properties. The modification of SMA properties can be caused by the temperature and/or stress. In the paper the new model of pseudoelascity with internal sub-loops is proposed. This model does not describe the physics of the process, but allows to approximate experimental data especially when the stress range is between minimum and maximum value. The presented approach is able to describe the turning points of internal hysteresis sub-loops. In the full conference paper, the model of SMA element (prosthesis) will be inserted in the model of the human middle ear to reconstruct a damaged ossicular chain.

a)



b)

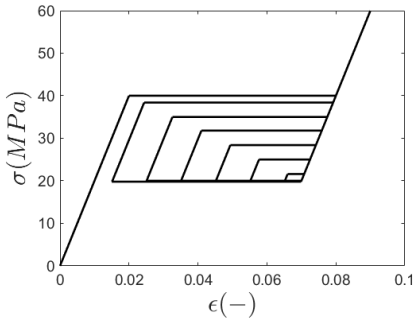


Figure 3: Internal hysteresis loops (second case): a) time series of strain, b) stress versus strain.

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