Finite element and plasticity models in analysis of confine concrete column

Artur Kotarski[1] **and Zdzisław Więckowski**[2]

[1] *Faculty of Civil, Architectural and Environmental Engineering, Łódź University of Technology*
Al. Politechniki 6, 90-924 Łódź, Poland
e-mail: Artur.Kotarski@p.lodz.pl

[2] *Faculty of Civil, Architectural and Environmental Engineering, Łódź University of Technology*
Al. Politechniki 6, 90-924 Łódź, Poland
e-mail: Zdzislaw.Wieckowski@p.lodz.pl

Abstract

The mechanical behaviour of a confine concrete column has been studied by the finite element method. An effective two-dimensional approach has been proposed. Several constitutive models based on plasticity theory have been utilised to describe the material properties of concrete. The obtained numerical results have been compared to experimental ones.

Keywords: finite element method, elastic–plastic material model, spiral reinforced column, confine concrete

1. Introduction

There is a number of papers where the problem of compression of a concrete column with lateral confining reinforcement was investigated numerically. For instance, the finite element method was employed in [3, 1] where the problem was treated as three-dimensional. Such a 3D analysis is time consuming and requires some significant effort related to definition of a computational element mesh. The effort can be made smaller by reducing the problem to the two-dimensional one. In the case of compression of a prismatic column, the stress state in a cross-section of the column can be considered as independent of a position of the section along the column. This means that the stress field is a solution of the generalized plane strain problem with given contraction of the column which plays a role of a load. Such a 2D approach was proposed in [2] where the concrete core of the column was treated in a rather simply way as an elastic–plastic body with the Drucker–Prager yield condition and the flow rule with no dilation. More realistic elastic–plastic models have been applied in the present work, namely, two non-associative models with non-zero plastic dilation and the Drucker–Prager and Mohr–Coulomb plastic conditions, and the associative Willam–Warnke model. Several types of a column's cross-section have been analysed with various patterns of lateral reinforcement like spirals or stirrups. To describe the reinforcement, the elastic–perfectly plastic material model with the Huber–von Mises yield condition has been used. The finite element interpolation has been done with the parabolic triangular elements for discretising the concrete core and the parabolic bar elements for representing the reinforcement. The influence of the cross-section part representing concrete outside the lateral reinforcement has been neglected as spalling is expected when column's compression has advanced.

2. Setting of the problem

A prismatic concrete column with a uniformly distributed confining reinforcement is considered. If x_3-axis is oriented along the column, the strain state in the column can be described by three unknown non-zero components $\varepsilon_{\alpha\beta}$ $(\alpha, \beta = 1, 2)$ with one given $\varepsilon_{33} = -E_{33}$ where E_{33} denotes the longitudinal shortening of the column. Four components of the stress tensor are to be found: $\sigma_{\alpha\beta}$ and σ_{33} $(\sigma_{3\alpha} = 0)$. The main purpose of the analysis is to find the resulting axial force in the column which can be expressed by the average stress σ_{33} in concrete and the longitudinal reinforcement.

Three elastic–perfectly plastic constitutive relations have been utilized to describe the behaviour of concrete. The relations can be written in the following form:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^{\mathrm{e}}_{ij} + \dot{\varepsilon}^{\mathrm{p}}_{ij}, \quad \dot{\varepsilon}^{\mathrm{e}}_{ij} = C_{ijkl}\,\dot{\sigma}_{kl},$$
$$\dot{\varepsilon}^{\mathrm{p}}_{ij} = \begin{cases} \dot{\lambda}\,\dfrac{\partial g}{\partial \sigma_{ij}} & \text{if } f(\sigma_{ij}) = 0 \quad \text{with } \dot{\lambda} \geq 0, \\ 0 & \text{if } f(\sigma_{ij}) < 0, \end{cases} \tag{1}$$

where a dot denotes the time derivative, ε^{e} and ε^{p} are the elastic and plastic components of the strain tensor, respectively, C_{ijkl} denotes the tensor of elastic compliance, f the plastic yield function, and g the plastic potential defining the flow rule. Two non-associative models with the Mohr–Coulomb and Drucker–Prager yield conditions have been applied for which $f_{\mathrm{MC}}(\varphi) = -p\,\sin\varphi + q\,(\cos\Theta/\sqrt{3} - \sin\varphi\,\sin\Theta/3) - c\,\cos\varphi$, and $f_{\mathrm{DP}}(\varphi) = q - m\,p - k$, respectively, where $p = -\sigma_{ii}/3$, $q = \sqrt{3/2\,s_{ij}\,s_{ij}}$ with $s_{ij} = \sigma_{ij} + p\,\delta_{ij}$, $m = (6\,\sin\varphi)/(3 + \sin\varphi)$, $k = (6\,c\,\cos\varphi)/(3 + \sin\varphi)$ and $\Theta = 1/3\,\arcsin(-27/2\,J_3/q^3)$ with J_3 denoting the third invariant of the stress deviator. In the above definitions, φ is the angle of internal friction and c the cohesion. The plastic potentials can be expressed as follows: $g_{\mathrm{MC}} = f_{\mathrm{MC}}(\psi)$, $g_{\mathrm{DP}} = f_{\mathrm{DP}}(\psi)$ where ψ is the dilatancy angle.

The associative Willam–Warnke model has also been applied in the analysis. The yield function in this model is only briefly expressed here, $f_{\mathrm{WW}} = q - \sqrt{15/2}\,f'_{\mathrm{c}}\,r(p, \Theta)$ where f'_{c} denotes the compressive strength of concrete and r the function depending on pressure p and Lode's angle Θ [6].

3. Finite element solution

The problem can be formulated in the variational form using the equation of virtual work

$$\int_{\Omega} \sigma_{ij}\,\delta\varepsilon_{ij}\,\mathrm{d}\Omega + \oint_{S} A\,\sigma\,\delta\varepsilon\,\mathrm{d}s = 0 \quad \forall \delta\boldsymbol{u} \in V_0 \tag{2}$$

where S is a line representing the axis of a reinforcement, A the area of a reinforcement cross-section, σ and ε denote the normal stress and longitudinal strain in the reinforcement. $\delta\boldsymbol{u}$ denotes a variation of the displacement field which belongs to the space of kinematically admissible displacement fields, V_0, that are sufficiently regular and satisfy the homogeneous kinematic boundary conditions.

After using the finite element interpolation for the displacement field by means of 6-node triangular elements for region Ω and 3-node bar elements for the reinforcement, Eq. (2) takes the following form:

$$\int_{\Omega} \mathbf{B}^{\mathrm{T}} \boldsymbol{\sigma} \, \mathrm{d}\Omega + \oint_{S} A \, \bar{\mathbf{B}}^{\mathrm{T}} \sigma \, \mathrm{d}s = \mathbf{0} \tag{3}$$

where $\mathbf{B}$ and $\bar{\mathbf{B}}$ are the stress–displacement matrices for the plane strain triangular and bar elements, respectively, and $\boldsymbol{\sigma}$ denotes the stress vector, $[\sigma_{11} \; \sigma_{22} \; \sigma_{12}]^{\mathrm{T}}$. As in both the cases of plane strain and bar elements, the quadratic interpolation functions are used, the condition of continuity of displacement field has been satisfied on the concrete–reinforcement interface.

The equation system (3) is solved in an incremental way by increasing the magnitude of longitudinal strain component ε_{33} equivalent to column contraction taken with the negative value, $-E_{33}$. The contraction plays a role of loading. For each load increment, the system of non-linear equation (3) is solved iteratively by means of the modified Newton–Raphson method. The implicit procedure ([5, 8]) is employed to calculate the stress tensor satisfying the constitutive relations (1).

4. Example

The results obtained for two square cross-sections with five and four spirals, investigated experimentally in [7], have been demonstrated in the paper. In the case of the first section, the diameter of the central spiral has been set to 420 mm and the corner ones to 105 mm with the pitch 50 mm. The diameters of the spirals in the latter section and their pitch have been set to 360 mm and 75 mm, respectively. The thickness of all the spirals has been set to 13 mm. The cross-sections of the columns and the space discretisations of their quarters are shown in Fig. 1.

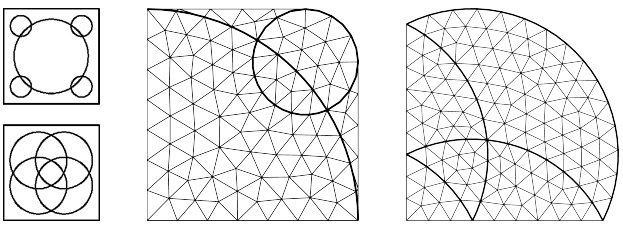

Figure 1: Cross-sections of analysed columns and triangulations

Computations have been made with the following material data for concrete: Young's modulus 30 GPa, Poisson's ratio 0.2, compressive strength 34.4 MPa, and for steel: Young's modulus 210 GPa and yield limit 274.7 MPa. To apply the Mohr–Coulomb and Drucker–Prager plasticity models, the value of the internal friction angle has been set, $\varphi = 37^{\circ}$ as indicated in [4], with the cohesion value $c = 8.551$ MPa.

The results of the computations have been shown in Fig. 2 in the form of relations between the contraction of the column and the average compressive concrete stress (as done in [7]). The left plot presents the relation obtained in the case of the the five-spiral cross-section while the right one is related to the four-spiral cross-section. The results of the case of plastically incompressible Drucker–Prager model ($\psi = 0$) has also been presented in the figure.

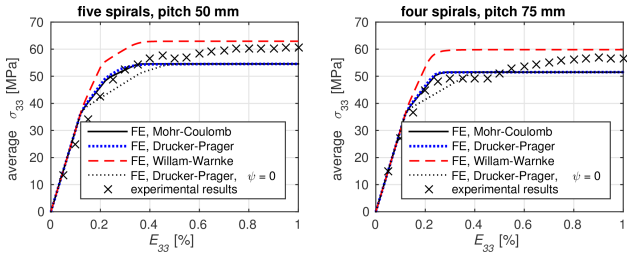

Figure 2: Averaged concrete stress–longitudinal strain relation

As can be seen in Fig. 2, the Willam–Warnke model has produced higher values of the load limit than the Mohr–Coulomb and Drucker–Prager constitutive relations. It can be noticed that the load limits of the two analysed column cross-sections obtained experimentally have placed between the numerical outcomes.

5. Conclusions

A two-dimensional finite element approach has been proposed for an analysis of the stress state in the cross-section of a confined concrete column. Three constitutive models of concrete have been utilised in the paper. The sought load limit has appeared to be significantly higher when the Willam–Warnke model was used in comparison to the Mohr–Coulomb and Drucker–Prager non-associative models. The numerical outcomes have been compared to the experimental ones. A fair agreement between both the results has been obtained.

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