Equilibrium paths analysis of materials with rheological properties using the chaos theory

Paweł Bednarek[1] **and Jan Rządkowski**[2]

[1]*Faculty of Civil Engineering, Wrocław University of Science and Technology*
wybrzeże Stanisława Wyspiańskiego 27, 50-370 Wrocław, Poland
e-mail: pawel_bednarek@o2.pl

[2]*Faculty of Civil Engineering, Wrocław University of Science and Technology*
wybrzeże Stanisława Wyspiańskiego 27, 50-370 Wrocław, Poland
e-mail: jan_rzadkowski@poczta.onet.pl

Abstract

The numerical equilibrium path analysis of the material with random rheological properties using standard procedures and specialist computer software has not been successful. The proper solution for the analysed material heuristic model was obtained on the base of the chaos theory elements and neural networks. The paper deals with mathematical reasons of the unsuccessful use of the computer software. The properties of the attractor used in analysis were also elaborated. It presents the results of conducted numerical analysis both in numerical and in graphical form for the used procedures.

Keywords: equilibrium path, viscoelastic material, Laplace transform, Rządkowski attractor, genetic algorithm

1. Introduction

Construction materials widely used in civil engineering structures i.e.: plastics, steel in high service temperature $T > 200^\circ C$, soil, etc. are characterised by some rheological properties like creeping and relaxation. After performing experimental creep and stress–relaxation test, data fitting to appropriate viscoelastic material model must be necessarily performed.

2. Numerical experiment data

Table 1: Rheological testing results for material parameters: $\{G_1, G_2, \eta_1, \eta_2\} = \{10, 2, 5, 1\} \cdot 10^3$ and model ref. (Fig. 1)

time	**creep test**	**stress–relaxation test**
$t\ [s]$	$\gamma(t)$	$\tau(t)$
0.0625	0.00003448388	4500.597
0.125	0.00006361715	4451.076
0.25	0.0001092349	4366.245
0.5	0.0001665065	4241.443
1.	0.0002161763	4104.931
2.	0.000242199	4019.819
4.	0.0002494235	4000.707
8.	0.0002499966	4000.001
16.	0.00025	4000.

Authors numerically obtained results for known viscoelastic material model presented in the Fig. 1. Creep and stress–relaxation tests were performed, results are given in the Tab. 1.

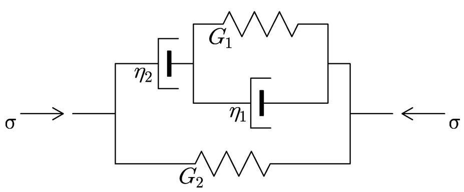

Figure 1: Rheological model of material

3. Fitting parameters of rheological material model

For material model presented in the Fig. 1 the formulas of compatibility for the material parameters G_i and η_i were calculated symbolically using the Laplace transform method [5, 6]. Having performed the rheological tests, basing on their results and symbolical math formulas, authors tried to perform reverse data fitting using residual methods.

Data fitting for stress–relaxation test has failed. Alerts indicate that decreasing iteration step of the parameters do not cause decreasing of the gradient (in a manner of the derivative of the trial function). In consequence such situation had to alter solution convergence deficiency. To confirm that ascertainment the differentiation of function $\tau(G_1, G_2, \eta_1, \eta_2, t)$ parameters has been carried out. For certain parameters: $\{G_1, G_2, \eta_1, \eta_2, t\} = \{G_1, 1, 1, 1, 1\}$ the trajectories of obtained derivatives have been analysed.

$$\lim_{G_1 \to -\infty} \frac{d\,\tau(G_1, G_2, \eta_1, \eta_2, t)}{d\,G_1} = \infty$$

In such case basic requirements of curve fitting methods based on residuals are not fulfilled.

Consequently to confirm the correctness of the rheological model of material it was decided to conduct the relaxation test parameters fitting using genetic algorithm method. Authors implemented genetic curve fitting algorithm based on principles given in [3]. Least squares method was taken as a fit quality criteria. The following parameters of GA were used:

Table 2: Genetic algorithm parameters

parameter	**value**
population size	128
number of unknowns	4
maximum generation	600
crossover probability	0.70
mutation probability	0.01
scaling	2.0

The genetic algorithm with parameters included in the Tab. 2 was used as a data fitting method for stress–relaxation test results. At the first step the range of parameters variabilities (the space of analysis) was determined as:

$\{G_1\} \div \{1\text{E}4 \pm 500\}, \{G_2, \eta_1, \eta_2\} \div \{2\text{E}3, 5\text{E}3, 1\text{E}3\} \pm 100$

After averaging results from multiple algorithm execution (results from each exec. are equally probable) method characterizes with very fast convergence to material parameters referred in the Tab. 1.

Taking under consideration necessarity of using functions obtained from the Laplace transform method [5], it appears that in real life material testing it is very difficult to find appropriate rheological material model based on elastic and viscous elements. As an alternative, authors proposed using so called Rządkowski's attractor [4] in coincidence with neural network curve fitting algorithm.

4. Material model analysis using the chaos theory

In the chaos theory the simplest way of the equilibrium paths approximation is to apply the so-called Rządkowski's attractor [4]. The attractor is a stable set of lines approached by all trajectories of the exponential functions describing the relationships.

$$\delta = f(Q) = \alpha \cdot Q^{\alpha-1} \tag{1}$$

In the Eq. (1) Q is a generalised load and δ is a generalised deformation of structure built from materials with elastic, plastic, viscous or brittle features. The normalized base graphic form of the attractor is presented in the Fig. 2 where α is the so called driving parameter.

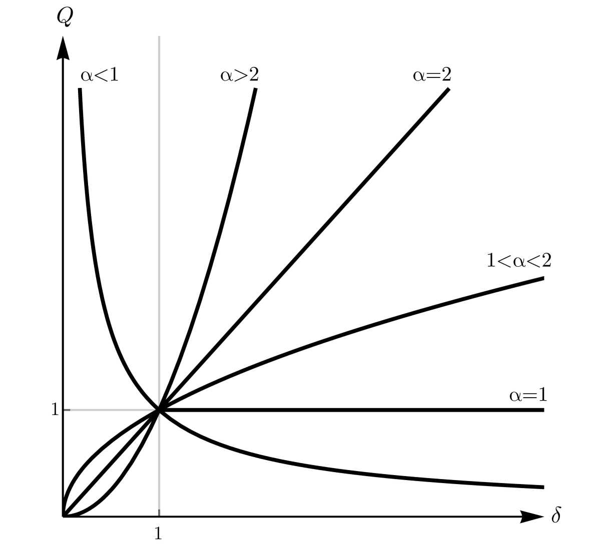

Figure 2: Normalized base form of the Rządkowski's attractor

The base attractor branch, corresponding to the values of the steering parameter $1 < \alpha < 2$ is mapping the equilibrium paths' trajectories of the elastic and elastic-plastic structures with material and geometric non-linearity. The attractor branch, corresponding to the values of the steering parameter $0 < \alpha < 1$, maps the equilibrium paths' trajectories of a structures for the rheological phenomena connected with creeping. For the values of the steering parameter $\alpha = 1$ the attractor branch is mapping the equilibrium path $\alpha = 2$ trajectory of the ideal elastic-plastic Prandtl's material, for $\alpha = 1$ is mapping an ideally elastic material model, however for $\alpha > 2$ is mapping the phenomena analogical to rheopexy. The attractor point determined with the coordinates $\{Q = 1, \delta = 1\}$ can be regarded as a criterion for the limit state of the structure. In the event of a complicated form of the equilibrium path, the exponent α can be presented in the form of a function, and instead of the multiplier α the multiplier β (directional coefficient) can be entered:

$$\delta = \beta \cdot Q^{f(\alpha)-1} \tag{2}$$

The function (2) can also be used transformed by relatedness to the form given by Eq. (3).

$$\delta = \beta \cdot Q^b e^c \tag{3}$$

The results of creeping test conducted using the proposed algorithm based on the Rządkowski attractor and curve fitting algorithm based on neural networks according to [1] were coincided with very high accuracy with the results obtained by using the tests of compatibility. The result of stress–relaxation approximation is presented in the Fig. 3.

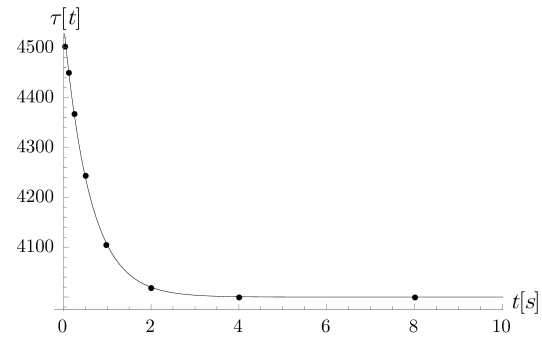

Figure 3: Graph of the stress–relaxation test for parameters $\{G_1, G_2, \eta_1, \eta_2\}$ given in Tab. 1

5. Conclusions

In equilibrium path analysis of the materials' properties the conjunction of presented attractor and neural networks gives much better results, than using polynomial function [2]. Obtained solutions are always unequivocal and there is no necessity for additional interpretation of obtained numerical results especially in estimation of material parameters dependent on time t like the value of deformations $\delta(t)$ or displacements $\Delta(t)$. This concerns both the viscoelastic and elastic-plastic materials as well as structures built from these materials.

References

[1] Groupe, D., *Principles of Artificial Neural Networks*. 2nd edn.,World Scientific Publishing, Hackensack NY, 2007.

[2] Leshno, M., Lin, YA., Pinkus, A. and Schockem, S., Multilayer feedforward networks with a polynomial activation function can approximate any function, *Neural Networks 6*, pp. 861-867, 1993.

[3] Reeves, C. and Rowe, J.E., *Genetic Algorithms: Principles and Perspectives: A Guide to GA Theory*, Springer US, 2002.

[4] Rządkowski, J. and Kirianova, L., Limit states of engineering structures in the aspect of chaos theory, *Procedia Engineering 153*, pp. 283-286, 2007.

[5] Schiff, JL., *The Laplace Transform. Theory and Application*, Springer-Verlag, NY, 1999.

[6] Tschoegel, NW., Time Dependence in Materials Properties: An Overview, *Mechanics of Time Dependent Materials*, Vol. 1, pp. 3-31, 1997