Effective elastic properties of sintered materials with branched cracks

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Abstract

The aim of the work is analysis of sintered materials with branched cracks growing from the voids situated at corners of fibers. The material is modelled as a two-dimensional linear-elastic structure using the boundary element method (BEM). The materials without voids and with voids having different shapes are considered. The influence of lengths of the cracks and shape of the voids on stress intensity factors (SIF) and effective elastic properties (the Young modulus and Poisson ratio) are studied. The overall properties of the sintered material are determined by considering the representative volume element (RVE) with large number of branched cracks.

Keywords: sintered material, branched cracks, boundary element method (BEM), representative volume element (RVE), effective properties, stress intensity factors (SIF)

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1. Introduction

The powders can be formed into the required shape by applying high pressure. During the pressing the particles are deformed plastically. After pressing the material is subjected to sintering, which is a heat treatment operation, in order to bond metallic particles and increase strength. During sintering the metal remains unmelted. The typical compaction pressure is 700 MPa, temperature of sintering is 1200 °C and time is 45 minutes for steel powders [5]. The features of the sintered material depend on properties of powder particles and their interaction. The unidirectional fibres hexagonally packed subjected to pressing during plastic deformation transform into hexahedral rods. The material contains small pores at the corners of fibres with cracks along boundaries of fibres.

Olevsky [6] presented a continuum mechanics approach to the solution of technological problems of sintering taking into account thermo-mechanical aspects of the process.

Borovik [1] determined the effective elastic properties of a sintered material with pore channels treated as Y-shaped cracks, which create a periodic regular hexagonal network. A unit cell containing halves of two adjacent cracks was modeled using the finite element method. The influence of different dimensions of cracks branches on the effective Young modulus and the Poisson ratio was investigated. Borovik [2] for the same material and the computer model analyzed stress intensity factors (SIF) at the tips of branched cracks. The material was subjected to the uniaxial tension and the influence of lengths of crack arms on SIF was studied. Daux et al. [3] presented the extended finite element method (X-FEM) to analysis of cracks with multiple branches and cracks emanating from holes. A standard displacement approximation was enriched by incorporating additional discontinuous functions. The method allows the modelling of discontinuities independently of the mesh. Sevostianov et. al [8] studied the influence of the microstructure of the sintered metal fibers on elastic properties. The analytical methods: non-interaction approximation, effective media approach, differential approach and effective field methods were used to derive explicit formulas for calculation of effective properties. The effect of relative volume of pores, pore shapes and relative length of crack branches, which depend on the temperature of sintering, was analyzed.

In the present work a sintered metal fibres with voids and branched cracks grooving from their centres are analyzed. The fibres are considered as a continuous matrix. The problem is treated as a two-dimensional by considering the cross section of the material perpendicular to the fibres. In the papers presented by other authors the sintered material was analyzed by considering a single branched crack or a small unit cell with parts of the cracks. In the present paper the representative volume elements (RVE) containing large number of regularly distributed branched cracks are investigated. The influence of void shapes and dimensions of the crack branches on stress intensity factors and elastic properties are studied. The computed results are compared with available solutions presented in the literature. In the next section an example of the RVE with branched cracks is given.

The microcracks in linear-elastic, isotropic and homogenous RVE are analyzed using the dual boundary element method (DBEM) [7]. In this approach only boundaries of the body and crack surfaces are divided into boundary elements. The variations of boundary coordinates, displacements and tractions are interpolated using shape functions and nodal values. In the DBEM the relations between boundary displacements and tractions are expressed by the displacement and traction boundary integral equations. The DBEM allows generation of complex RVE more easily than the FEM and gives very accurate results for crack problems.

The method was applied by Fedeliński [4] for computation of effective elastic properties and an analysis of stress intensity factors for representative volume elements with randomly distributed microcracks. The microcracks having the same length, randomly distributed, parallel or randomly oriented were considered. The influence of density of microcracks on the effective Young modulus, the effective Poisson ratio and stress intensity factors was presented. The numerically computed Young moduli were compared with the solutions obtained by analytical methods: the non-interacting method, the self-consistent method and the differential method.

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* The scientific research is financed by National Science Centre, Poland in years 2016-2019, grant no. 2015/19/B/ST8/02629.

2. Numerical example

An example of the representative volume element of sintered material containing 28 branched cracks is presented in Fig. 1. The cracks are situated at the corners of sintered fibers. The magnified cracks are shown in Fig. 2. The lengths of the branches are a and the distance between crack centres is l, where a/l=0.25. The length of the cracks in the horizontal direction is $2c_1$ and in the vertical direction $2c_2$. The plate is in plain strain conditions and the Poisson ratio is ν=0.3. The plate is subjected to the uniformly distributed horizontal t_1 or vertical tractions t_2. The plate is divided into 752 boundary elements (8 elements for each crack branch and 80 for the external boundary). The initial and the deformed shape of the RVE for the vertical loading are presented in Fig. 3.

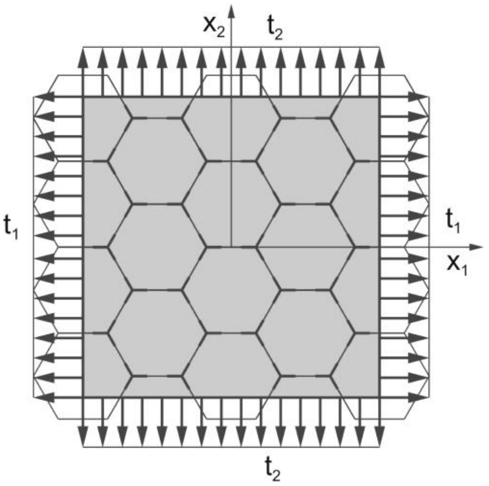

Figure 1: RVE of the sintered material containing 28 branched cracks

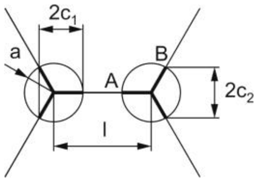

Figure 2: Dimensions of the branched cracks

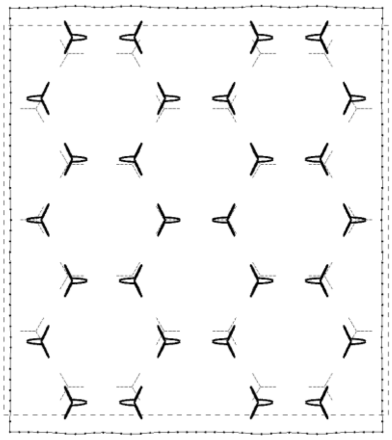

Figure 3: Initial (dashed line) and deformed shape (continuous line) of the RVE for the vertical loading

The stress intensity factors are computed for all cracks. The SIF for the horizontal loading are normalized with respect to the SIF $K_o=t_1(\pi c_2)^{1/2}$ and for the vertical loading with respect to the SIF $K_o=t_2(\pi c_1)^{1/2}$. The normalized SIF for the branched crack in the centre of the RVE are given in Table 1.

Table 1: Normalized stress intensity factors

loading	tip A		tip B	
	K_I/K_o	K_{II}/K_o	K_I/K_o	K_{II}/K_o
horizontal	0.029	0.000	0.867	0.590
vertical	1.234	0.000	0.332	0.635

The displacements of the external boundaries of the RVE are used to compute average strains in the horizontal and vertical direction and the effective Young moduli E_1, E_2 and Poisson ratios ν_{12}, ν_{21}. The effective properties are normalized with respect to the properties of the plate without cracks E_o, ν_o and are given in Table 2. The present effective Young moduli agree well with the results presented by Grabovik [1] and Sevostianov et. al [8].

Table 2: Effective material properties

E_1/E_o	E_2/E_o	ν_{12}/ν_o	ν_{21}/ν_o
0.793	0.783	0.822	0.841

3. Conclusions

The boundary element method allows simple and accurate computation of stress intensity factors and effective properties of sintered materials. The computed effective elastic properties agree well with the available results presented in the literature. Contrary to the methods given in the literature the present results are obtained for large number of fibres and cracks.

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