Problem of FGM TBC coated cylinder

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Abstract

This paper demonstrates the influence of several approximations of FGM thermal barrier coatings on the temperature and stress distributions. It is shown that the conventional approximations by specific power or exponential functions reflect FGM distributions coming from neither theory nor practise. As example, a thick-walled cylinder including FGM interface subjected thermo-mechanical loading is considered. Two independent formulations of the uncoupled thermo-elastic problem based on the displacement or stress function are tested.

Keywords: functionally graded material, thermal barrier coating, thermo-elasticity, thick-walled cylinder

1. Introduction

Functionally graded materials (FGMs) provide thermal insulation and mechanical toughness at high temperature by varying the composition of thermal conductivity coefficient, thermal expansion coefficient and Young's modulus from high temperature side to low temperature side continuously and simultaneously by removing the discontinuity of layered structures. Numerous analytical solutions of thermo-elastic plane or three-dimensional problems of FGMs take advantage of specific power or exponential function approximation methods of multi-layered composite structures (cf. Batra [1]). Hence, question how to model real gradation of thermo-mechanical properties seems to be important.

2. General formulation of FGM thermo-elastic cylinder

A thermo-elastic rotationally-symmetric cylinder including FGM interface is considered. The cylinder is established a temperature field $T+\theta(r)$, where T in the temperature of the solid corresponding to zero stress and strain, as well as internal pressure p (see Fig. 1).

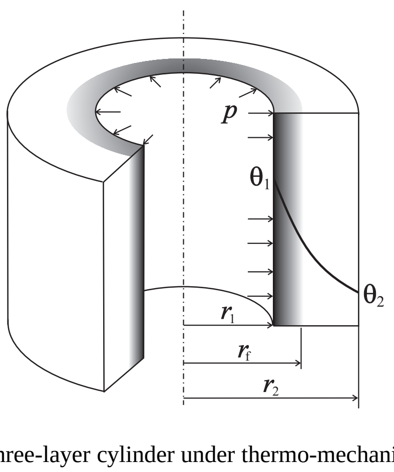

Figure 1: Three-layer cylinder under thermo-mechanical loading

The system of equations of uncoupled thermo-elasticity expressed in displacements is as follows

$$\begin{aligned} &u'' + \left(\frac{1}{r}+\frac{E'}{E}\right)u' + \left(\frac{\nu}{1-\nu}\frac{E'}{E}-\frac{1}{r}\right)\frac{u}{r} = \frac{A}{1-\nu}\frac{(E\alpha\theta)'}{E} \\ &\theta'' + \left(\frac{\lambda'}{\lambda}+\frac{1}{r}\right)\theta' = 0 \end{aligned} \tag{1}$$

or alternatively by stress function formulation

$$\begin{aligned} &F'' + \left(\frac{1}{r}-\frac{E'}{E}\right)F' + \left(\frac{\nu}{1-\nu}\frac{E'}{E}-\frac{1}{r}\right)\frac{F}{r} = -\frac{BE}{1-\nu}(\alpha\theta)' \\ &\theta'' + \left(\frac{\lambda'}{\lambda}+\frac{1}{r}\right)\theta' = 0 \end{aligned} \tag{2}$$

where formats of constants A and B depend on plane strain state type according to following scheme: in case of plane strain state imposed on both mechanical and thermal deformation $A=1+\nu, \quad B=1$, whereas in case of plane strain state imposed on thermal deformation only $A=1, \quad B=\frac{1}{1+\nu}$. Stress components as well as heat flux are expressed by conventional relations

$$\begin{aligned} &\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)u' + \nu\frac{u}{r} - A\alpha\theta\right] = \frac{F}{r} \\ &\sigma_\varphi = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{u}{r} + \nu u' - A\alpha\theta\right] = F' \\ &q = -\lambda\theta' \end{aligned} \tag{3}$$

3. Modeling of FGM interface

All thermo-mechanical properties of the FGM interface such as α, λ and E are arbitrary functions of radius r. Following five approximations are considered (see Fig. 2): benchmark problem of a ceramic layer deposed directly on metallic substrate

$$f_1(r) = f_\mathrm{c} \tag{4}$$

conventional power function of interface

$$f_2(r) = (f_\mathrm{m}-f_\mathrm{c})\frac{r^n}{r_\mathrm{f}^n - r_1^n} \quad \text{where} \quad n = \frac{\log(f_\mathrm{m}/f_\mathrm{c})}{\log(r_\mathrm{f}/r_1)} \tag{5}$$

linear function interface

$$f_3(r) = (f_\mathrm{m}-f_\mathrm{c})\frac{r-r_1}{r_\mathrm{f}-r_1} + f_\mathrm{c} \tag{6}$$

tangent hyperbolic (one step smooth) function of interface (cf. Ganczarski and Szubartowski [2])

$$f_4(r) = \frac{f_\mathrm{m}+f_\mathrm{c}}{2} + \frac{f_\mathrm{m}-f_\mathrm{c}}{2}\tanh(ar+b) \tag{7}$$

multiple tangent hyperbolic (multi-step smooth) function of interface

$$f_5(r) = \frac{f_\mathrm{m} + f_\mathrm{c}}{2} + \frac{f_\mathrm{m} - f_\mathrm{c}}{2} \sum_{i=1}^{n=4} \tanh(a_i r + b_i) \qquad (8)$$

where $f_i(r)$ stand for respective property $\alpha(r)$, $\lambda(r)$ or $E(r)$, indices "c" and "m" refer to ceramic or metallic materials, parameters a_i and b_i define location and thickness of interface sub-layer.

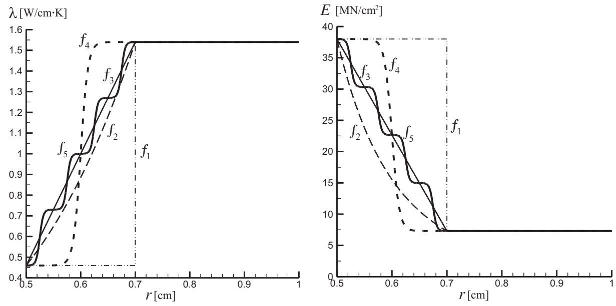

Figure 2: Distributions of thermal conductivity $\lambda(r)$ and Young's Modulus $E(r)$

4. Example

A system of four first-order ordinary differential equations corresponding to (1) or (2) is numerically integrated from $r_1 = 0.5$ cm to $r_2 = 1.0$ cm by use of shootf.for routine, being shooting to a fitting point $r_\mathrm{f} = 0.7$ cm implementation of multidimensional, globally convergent Newton-Raphson method (cf. Press et al. [3]). Thermo-mechanical boundary and continuity conditions are assumed as follows:

$$\begin{array}{lll} \theta(r_1) = \theta_1, & q_\mathrm{c}(r_\mathrm{f}) = q_\mathrm{m}(r_\mathrm{f}), & \theta(r_2) = \theta_2 \\ \sigma_{r\mathrm{c}}(r_1) = -p, & \sigma_{r\mathrm{c}}(r_\mathrm{f}) = \sigma_{r\mathrm{m}}(r_\mathrm{f}), & \sigma_{r\mathrm{m}}(r_2) = 0 \end{array} \qquad (9)$$

where $p = 0.55$ kN/cm^2, $\theta_1 = 25°$C and $\theta_2 = 0°$C. Complete material data of both materials constituents of FGM, after Wang et al. [4], are presented in Table 1.

Table 1: Selected properties of constituents of FGM after Wang et al. [4]

Constituent	E	ν	λ	$\alpha \cdot 10^{-6}$
	[MN/cm^2]	[–]	[W/cmK]	[1/K]
Al substrate	7.3	0.3	1.54	23
Al_2O_3 coating	38	–	0.46	8.5

Temperature distributions corresponding to all five types of interface, described by Eqs (4-8), are shown in Fig. 3. In comparison with the temperature obtained for the benchmark problem (ceramic layer deposed directly over the metallic substrate), the temperature fields referring to subsequent types of interface represent smooth and monotonously decreasing functions, that are located rather close each to other although corresponding distributions of thermal conductivity essentially differ (see Fig. 2). This is a consequence of both Dirichlet's type of boundary conditions imposed on temperature Eq. (9_1) and simultaneously term λ'/λ, which is directly responsible for thermal inhomogeneity in Fourier's equation.

Solution of mechanical problem is illustrated by distribution of hoop stress, which is the dominant component of stress, in Fig. 4. Analogously to the temperature, in case of the benchmark problem, hoop stress exhibits strong discontinuity mainly caused by thermo-mechanical mismatch $E_\mathrm{c} \gg E_\mathrm{m}, \alpha_\mathrm{c} \ll \alpha_\mathrm{m}$. This inconvenient effect is successively eliminated by application of subsequent types of interface. Nevertheless, another inconvenient effect of tensile stress zone in ceramic layer occurs.

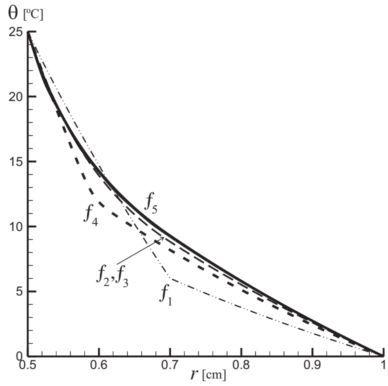

Figure 3: Distributions of temperature $T(r)$

As a consequence, a ceramic material of very low tensile strength is unable to carry tensile stress unless there exists residual stress in material/structure which is big enough to neutralize tensile hoop stress.

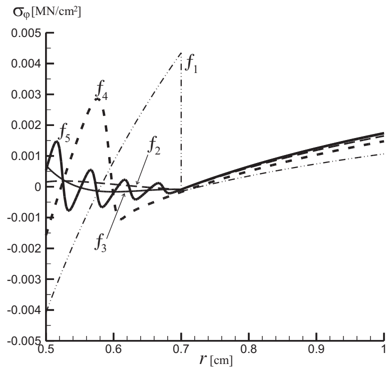

Figure 4: Distributions of hoop stress $\sigma_\varphi(r)$

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