## Multiobjective optimization of 3D porous thermoelastic structures

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## Abstract

This paper is devoted to the optimal design of pores in microstructures with respect to their thermal and mechanical properties. The porous material was modeled by means of two-scale analysis with numerical homogenization method. In order to model the shape of the pores, closed B-Spline surfaces were used. This allowed an optimization task in which the pores may have an almost arbitrary shape. The task of optimal design of the pores was performed with respect to mechanical, thermal and geometrical properties. Three different criteria, which depend on the stress value, the ability to conduct heat and porosity were defined. In-house implementation of a multiobjective evolutionary algorithm was used as an optimization tool. Numerical examples of optimization were included.

Keywords: porous materials, multi-objective optimization, evolutionary algorithms, thermoelasticity, multiscale modelling, finite element method, numerical homogenization

### 1. Introduction

In recent years a lot of effort has been made to design novel and smart materials, which require a combination of coupled field analyses, multiscale modelling and optimization methods.

In the case of structures under thermomechanical loading, both mechanical and thermal properties are to be optimized. For example, under a given thermomechanical load, to increase strength of the structure, reduced tensions must be lowered, to increase stiffness, displacements must be lowered and to increase thermal conductivity, temperature must be increased.

The analyst is required to define the proper functionals for the considered criteria in order to solve optimization tasks. It is very common that these functionals, derived from different physical fields (e.g. mechanical, thermal), are contradictory. These functionals for real life engineering problems are very often strongly multimodal. There is a need for an efficient multiobjective optimization method that is resistant to getting stuck in local minima [1]. In this work, optimization in twoscale thermoelastic problems by means of numerical homogenization and multiobjective evolutionary algorithms was considered. Properties of the microstructure (elastic and thermal constants) were calculated based on the objective functionals, which were numerically computed taking the quantities from the macro-scale into consideration. This work is an extension of the previous work in which optimization tasks have been solved for 2D structures and 3D structures with cylindrical inclusions [3, 4].

## 2. Formulation of the problem

In this work, a two-scale 3D thermomechanical model of porous structure was considered. Microstructures with local periodicity were considered and linear uncoupled thermoelasticity was assumed. The representative volume element (RVE) concept, modelled with periodical boundary conditions was considered [6]. Boundary-value problems for

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RVEs (micro-scale) were solved using finite elements method (FEM) to obtain macro-scale material parameters. Material properties such as Young modulus, Poisson ratio and thermal conductivity were homogenized. Multiple boundary-value problems for parametric models were solved using FEM software [10].

Design variables in multiscale optimization tasks were parameters describing size and shape of the void in the microstructure. B-splines were utilised to describe the void to achieve a high versatility of shapes. Differential equations of heat conduction and elasticity supplemented by mechanical and thermal boundary conditions described the linear thermoealasticity problem. In the fig. 1 an example of such a structure is presented.



Figure 1: Two-scale model of the 3D porous structure

#### 3. Multiobjective optimization algorithm

Evolutionary Algorithms are a group of bioinspired optimization methods which are resistant to getting stuck in local minima and do not require gradients of fitness functions to be calculated during the run of the algorithm.

The idea of multiobjective optimization is to optimize not a single criterion, but a vector of criteria at the same time, thus a result of such an algorithm run is a set of non-dominated solutions obtained for the contradictory criteria. These solutions are optimal in the Pareto sense. As evolutionary methods naturally process sets of solutions, they are convenient to be used for multiobjective optimization tasks.

In this paper, an in-house implementation of the multiobjective evolutionary algorithm based on the Pareto concept is used. Some ideas inspired by the NSGA-II algorithm are used to improve the multiobjective evolutionary algorithm [5]. The in-house implementation of the algorithm was tested both on benchmarks and real optimization problems and proven superior to NSGA-II, especially when dealing with difficult optimization tasks, involving strongly multimodal functions, non-convex or discontinuous Pareto fronts [2].

## 4. Designing the porous microstructure

For structures under thermomechanical loading, the optimization concerns both mechanical and thermal properties (e.g. strength, stiffness, low or high thermal conductivity). Multiscale optimization tasks were formulated as a design of a microstructure (shape of voids) for optimization criteria which are defined on the basis of quantities calculated for the macroscale [11]. The following objective functions were formulated: minimization of the equivalent stress in the macromodel, maximization of the heat flux of the structure and maximization of the porosity defined as the ratio of pore volume to the volume of RVE.

Two-scale models of porous structures with global periodicity are examined. As an example of such a micro-macro thermoelastic model, the cuboid solid made of porous aluminum, is considered. The macro-model is supplemented by thermal and mechanical loads.

The multiobjective optimization task concerns determining the size, shape and position of the cylindrical void in the microstructure by minimization or maximization of the functionals calculated on the basis of results obtained from the macro-model. In order to model the void in the microstructure B-Splines are used. Moreover, the void design parameters are responsible for the rotation and location of the void. The proposed parameterization allows modeling of the microstructure for free shape voids arbitrarily located and oriented [8]. Three exemplary models of the microstructure after discretization are shown in Fig. 2.



Figure 2: Examples of microstructure model

The microstructure is modeled as RVE with periodic boundary conditions. When solving thermoelasticity problems, numerical homogenization is utilized to compute the following effective constants: thermal expansion and heat conduction coefficients and elastic constants. There is no need to homogenize the thermal expansion coefficient as it is invariant for porous materials [9].

# 5. Representation of the results for multiobjective optimization tasks

Results of the multiobjective optimization tasks are sets of non-dominated solutions. Each solution is a vector of size equal to the number of criteria to be optimized. In the problem concerned in this paper there are 3 criteria which can be displayed as a 3D plot, each axis corresponding to a single criterion, and each point on the plot being a single nondominated solution. To enhance the clear, simple and precise manner of display of the multiobjective optimization problems some alternative methods can be utilized. These methods include representation of the results in the form of Self-Organizing Maps (SOM) and multiple-plot methods [7]. SOMs can be utilized to project *n*-dimensional Pareto fronts as coloured points on *n* two-dimensional maps and are remarkably useful when considering more than 3 criteria, which is often the case in real multiobjective optimization problems.

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