Numerical homogenization of inhomogeneous media with imprecise parameters

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Abstract

This paper is devoted to the numerical homogenization of porous materials with parameters uncertainty, represented by information granularity. The aim of the analysis is to obtain range of homogenized material properties reducing number of necessary homogenization calculation comparing with classic attitude. The uncertainties are represented by means of interval numbers. The directed interval arithmetic is employed to narrow the results range. Interval finite element method is employed to solve a boundary-value problem in micro scale. A numerical example presenting the efficiency of proposed attitude is attached.

Keywords: numerical homogenization, directed interval arithmetic, multiscale modelling, uncertainty analysis, porous materials

1. Introduction

Microscopically inhomogeneous materials, like porous materials or composites, are nowadays one of the most commonly used engineering materials. Different homogenization methods allow obtaining macroscopically homogenous equivalent material [6].

The certain material properties are usually not possible to obtain due to the production uncertainties, e.g. the manufacturing tolerances. To predict a possible range of material properties values, different methods considering various kinds of information granularity (interval numbers, fuzzy sets or stochastic variables) may be used [1]. In the present paper, an interval representation of uncertainties with the direct interval arithmetic attitude was employed.

The aim of the paper was to obtain reliable range of the equivalent elastic material properties values of porous material. Numerical homogenization was performed to obtain macroscopically equivalent properties of the media. The boundary-value problem was solved by means of the interval finite element method (FEM). The results obtained for different FEM meshes were compared.

2. Numerical homogenization

The homogenization procedure allows obtaining a medium macroscopically equivalent to a medium inhomogeneous in a micro scale. The numerical homogenization is one of the most efficient homogenization methods. It is assumed that the material is periodical and a representative volume element (RVE) fully represents the structure (global periodicity) or its part (local periodicity) [2].

The separation of scales condition states that the characteristic dimensions of RVE should be significantly larger than the characteristic dimensions at micro scale and considerably smaller than the macro-scale ones:

$$l_{micro} << l_{RVE} << l_{macro} \tag{1}$$

The following conditions must also be satisfied for RVE:

i) the Hill-Mandel condition for the equality of the average energy density at a micro scale and the macroscopic energy density at the point of macrostructure corresponding to the RVE:

$$\left\langle \sigma_{ij}\varepsilon_{ij} \right\rangle = \left\langle \sigma_{ij} \right\rangle \left\langle \varepsilon_{ij} \right\rangle \tag{2}$$

where: σ_{ij}, ε_{ij} – micro stress and strain tensors, respectively, V – the RVE volume, $\langle \cdot \rangle$ – the averaged value of the considered field:

$$\langle \cdot \rangle = \frac{1}{|V|} \int_V (\cdot) dV \tag{3}$$

ii) boundary conditions in the form of uniform traction conditions, uniform displacement conditions or periodic boundary conditions.

If the FEM method is applied to solve the boundary-value problem in both considered scales, the RVE must be assigned to each FEM integration point at the micro scale.

3. Directed interval arithmetic

Interval arithmetic is a method for solving problems with uncertain parameters [3]. The basis of the interval arithmetic is the interval representation of a single number:

$$\bar{a} = \left[a^-, a^+ \right] \tag{4}$$

Mathematical operations can be transferred to the interval number allowing to revive interval results. The classical version of interval arithmetic assumes that the value of a^- is always less than or equal to a^+, so that the interval is always determined in one direction.

Directed version allows other possibilities, introducing additional markings – functional of direction and functional of sign. The form of mathematical operations in the directed interval arithmetic depends on the value of the functionals of the interval numbers on which the operations will be performed. The main advantage of the directed interval arithmetic upon the classical one is that the obtained intervals are much narrower [4].

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Table 1: Numerical results of the homogenization

	Plate Young's modulus [GPa]		Homogenized Young's modulus [GPa] Case 1		Homogenized Young's modulus [GPa] Case 2	
Uncertainty level	Min. value	Max. value	Min. value	Max. value	Min. value	Max. value
0.1 %	199.9	200.1	123.0488	123.1933	122.8670	123.0128
0.5 %	199.5	200.5	122.7749	123.4973	122.5970	123.3254
1%	199.0	201.0	122.4614	123.9052	122.3000	123.7559
2%	198.0	202.0	121.9299	124.8111	121.8429	124.7466
5%	195.0	205.0	121.1586	128.2461	121.6342	128.7468

Each endpoint of the interval can be denoted as a^s, $s \in \{+,-\}$, where s is a binary variable. The variable s may be expressed as a product of two binary variables and it is defined as:

$$\begin{aligned} ++ &= -- = + \\ +- &= -+ = - \end{aligned} \tag{5}$$

An interval may be: proper for $a^- \leq a^+$, improper if $a^- \geq a^+$ and degenerate ($a^- = a^+$).

To solve the interval system of equations of the form:

$$\bar{A} \cdot \bar{x} = \bar{b} \tag{6}$$

different methods, like Rohn's method or combinatorial method, may be used [5]. In the present paper, the interval Gaussian elimination method was applied. In the first step, the main interval matrix $\bar{A}$ is decomposed to the product of the lower triangular interval matrix $\bar{L}$ and the upper interval triangular matrix $\bar{U}$. In the second step the system of equations of the form:

$$\bar{A} \cdot \bar{x} = \left(\bar{L} \cdot \bar{U}\right) \cdot \bar{x} = \bar{L} \cdot \left(\bar{U} \cdot \bar{x}\right) = \bar{b} \tag{7}$$

is solved by calculating the vector $\bar{z}$ from:

$$\bar{L} \cdot \bar{z} = \bar{b} \tag{8}$$

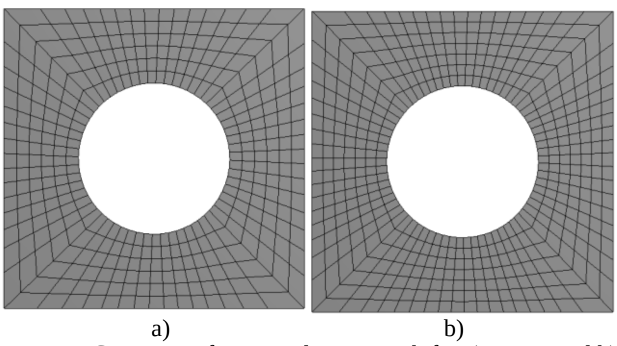
and vector $\bar{x}$ from:

$$\bar{U} \cdot \bar{x} = \bar{z} \tag{9}$$

4. Numerical example

A plate of dimensions 10x10 μm with centrally placed void was considered as representative volume element. The porosity of the structure was equal to 0.2.

The RVE was divided into 360 (case 1) and 483 (case 2) linear quadrilateral finite elements (Fig. 1), which resulted in systems of 420 and 552 equations, respectively.



a) b)

Figure 1: Geometry of RVE and FEM mesh for a) case 1 and b) case 2.

Displacement boundary conditions, with unit displacement values, were applied to fulfil the Hill-Mandel condition. Young's modulus of the plate material with mean value equal to 200 GPa was considered as uncertain parameter. Five different levels of uncertainty from the range 0.1÷5% were considered.

Homogenization problem was solved by means of interval FEM method. Interval values were introduced at the finite element stiffness matrix creation step. The interval values of the nodal reactions at the boundaries of the RVE were obtained as the solution of the system of equations.

Based on them, homogenized material properties were calculated. The results of the homogenization are collected in Table 1.

5. Final conclusions

Application of directed interval arithmetic for systems with uncertainties provide more efficient way of homogenization comparing to classical interval attitude, which results in very wide intervals. The resulting intervals obtained with interval MES are slightly higher than the actual values and their expansion increases with the increase in the number of equations in the solution being solved. On the other hand, reducing the number of equations would lead to narrower intervals but may reduce the accuracy of the calculated center value by insufficient discretization of the RVE.

The next stage of the research is to examine the influence of the use of many uncertain parameters on the homogenization results. Furthermore, other types of information granularity representation, e.g. fuzzy numbers, will be considered.

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