Numerical investigation of buckling of Cassini ovaloidal sandwich shells under external pressure

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Abstract

The paper deals with the buckling problem of three-layered shells of revolution with positive and negative Gaussian curvature under uniformly distributed external pressure. The shell is a sandwich structure composed of two isotropic facings and the core of metal foam. The meridian of the shell is a plane curve in the Cassini oval form. Geometrical properties of the middle surface of the shell are presented. The critical pressure of the analysed shells of different dimensions are numerically calculated with the use of the FEM in the ANSYS software. The results of these investigations are presented.

Keywords: shell of revolution, elastic buckling, metal foam core, external pressure, FEM analysis

1. Introduction

Stability of shells of revolution with different geometry of curve surfaces (e.g. spherical, ellipsoidal, ovaloidal, toroidal, etc) is one of the directions of recent research. In most of these papers is carried out research of single-layer shells subjected to external pressure. Ma *et al.* [4] studied the elastic buckling of ellipsoidal shells. The buckling of egg-shaped shells is described by Zhang *et al.* [8]. The problems of the buckling shells, e.g. clothoidal-spherical [2] and Cassini ovaloidal [3] are investigated by Jasion and Magnucki. A review of studies on the strength, static and dynamic stability of the non-conventional tanks for storage of liquid materials are presented by Zingoni [7]. Separate group of the research is dedicated to the stability of sandwich and multi-layered plates and shells. The main assessment criterion of application efficiency of these structures, except economic aspect, is relatively low mass-strength or mass-rigidity ratio. Strength and stability of the sandwich beams and plates with metal foam core are investigated by Magnucki and Szyc [5]. In this monograph, the experimental studies and modelling of the mechanical properties of these structures are presented. Belica *et al.* [1] studied the stability of cylindrical porous shells. Malinowski *et al.* [6] presented the results of research on buckling and post-buckling behaviour of elastic seven-layered cylindrical shells.

This paper is devoted to the problem of elastic buckling of three-layered Cassini ovaloidal shells of revolution subjected to uniformly distributed external pressure.

2. Model of Cassini ovaloidal shells

The wall of the shell consists of two isotropic facings and a core (Fig. 1). The core is made of a metal foam, its mechanical properties are isotropic. The geometry of the middle surface of the shell is defined as a Cassini oval curve. This curve in Cartesian coordinate system is defined in the following form:

$$y(x) = r(x) = \left[\sqrt{4c^2x^2 + a^4} - \left(c^2 + x^2\right)\right]^{1/2} \tag{1}$$

where: a and c are the parameters of the function, $r(x)$ is the radius of the parallel circle (Fig. 1).

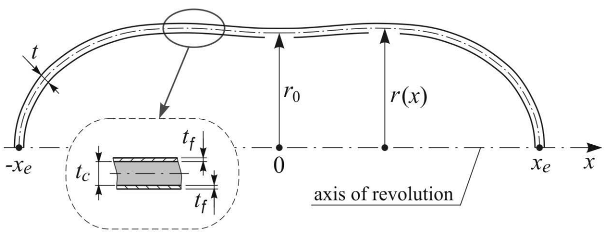

Figure 1: The Cassini ovaloidal sandwich shell

The value of the radius in the half length of the shell is equal to $r_0 = a\,(1 - k_c^2)^{1/2}$; the x-coordinate at the end of the shell equals $x_e = a\,(1 + k_c^2)^{1/2}$, where $k_c = c\,/a$ is a dimensionless parameter [3]. This parameter is characterised by the meridian shape of the shell. There are three possible cases:

- convex meridian, for $0 < k_c < 2^{-0.5}$;
- plano-convex meridian, for $k_c = 2^{-0.5}$;
- concavo-convex meridian, for $2^{-0.5} < k_c < 1$.

Exemplary shapes of these shells for different parameters k_c and Gaussian curvature K in the mid-length are shown in Figure 2.

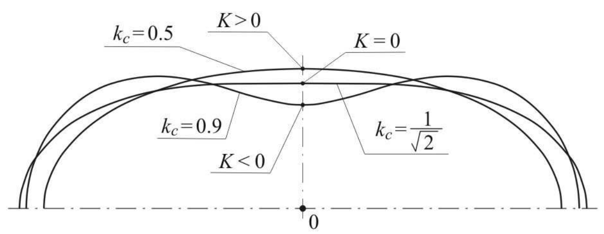

Figure 2: Cassini ovals of different meridians

The mass of the three-layer Cassini shell is defined by:

$$m_s = A_s\left(t_c\rho_c + (t - t_c)\rho_f\right) \tag{2}$$

where: t is the thickness of the shell wall; t_c is the thickness of the core; ρ_c and ρ_f are mass densities of the core and facings, respectively; A_s is lateral area of the shell

$$A_s = 4\pi\int_0^{x_e}\sqrt{1 + \left(\frac{dr}{dx}\right)^2}\, r(x)\,dx\ . \tag{3}$$

Table 1: Numerical results

k_c	0.5	0.55	0.6	0.65	$2^{-0.5}$	0.75	0.8	0.85	0.9
a [m]	1.1148	1.1291	1.1462	1.1668	1.1956	1.2220	1.2596	1.3071	1.3687
c [m]	0.5574	0.6210	0.6877	0.7584	0.8454	0.9165	1.0077	1.1110	1.2318
A_s [m^2]	14.302	14.383	14.493	14.640	14.869	15.096	15.441	15.902	16.520
t_f [mm]	2.774	2.755	2.731	2.699	2.650	2.603	2.534	2.447	2.338
r_0 [m]	0.9654	0.9430	0.9170	0.8867	0.8454	0.8083	0.7558	0.6886	0.5966
x_e [m]	1.2464	1.2886	1.3367	1.3916	1.4643	1.5275	1.6131	1.7155	1.8414
p_{cr} [MPa]	3.7785	3.1750	2.5687	1.9704	1.3333	0.9032	0.5520	0.3896	0.3311

The capacity of the shell is in the following form

$$V_s = 2\pi \int_0^{x_e} \left(r(x)\right)^2 dx \,. \tag{4}$$

3. Numerical calculation

Numerical analysis of the Cassini ovaloidal sandwich shells have been carried out with the use of finite element method (FEM) in the ANSYS software. The shells have been modelled using the shell element SHELL181 with four nodes and six degrees of freedom in each node. Three layers have been used in these elements: two faces of thickness t_f and the core of thickness t_c. Element SHELL181 is suitable for analysis of thin to moderately - thick shell structures [9]. The symmetry conditions have been applied in the symmetry plane of the shell - a half model has been obtained. Radial and circumferential boundary conditions have been applied to a small hole at the end of the shell. The sandwich shell has been subjected to uniform external pressure.

The solutions were carried out for the following constant data: $V_s = 5$ m^3, $m_s = 250$ kg; facings: $E_f = 65600$ MPa, $\rho_f = 2700$ kg/m^3; $\nu_f = 0.33$; the core: $t_c = 0.01$ m, $E_c = 1200$ MPa, $\rho_c = 250$ kg/m^3; $\nu_c = 0.34$. The design variables were: the dimensionless parameter k_c and the thickness of the facing t_f. The assumption of constant mass m_s and capacity V_s of the shell enables the calculation from equations (2) and (4) the thickness of the facings t_f and the parameter a.

The results of the calculations are presented in Table 1 and Figure 3. The smaller value of k_c, the higher the critical pressure a sandwich shell can carry. The buckling shape depends strongly on the geometry of the sandwich shell. For $k_c = 0.5$, $n = 10$; for $k_c = 2^{-0.5}$, $n = 6$ and for $k_c = 0.9$, $n = 2$, where n is the number of the circumferential waves.

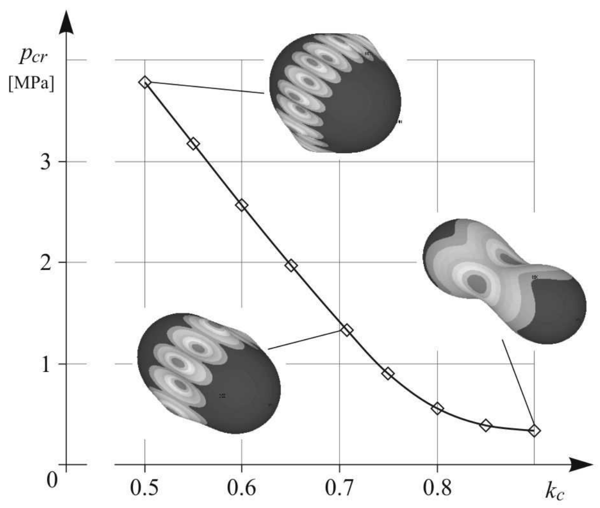

Figure 3: Critical pressures for Cassini ovaloidal shells

4. Conclusions

In the present studies, the stability of family shells subjected to uniformly distributed external pressure has been investigated. It is assumed that the meridian of mid-plane of sandwich shells of revolution is a curve in the Cassini oval form.

The research has been performed for constant value of the core thickness. Only buckling state of these shells was investigated. Further research will be directed to: study of the core thickness effect on critical pressure, analysis of the post-critical state (equilibrium path) of sandwich shells and comparison of numerical results for the three-layer and the single-layer shell.

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