Numerical analysis of thermomechanical low cycle fatigue

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Abstract

In this paper the numerical analysis of low cycle fatigue behaviour of steel in non-isothermal conditions is presented. First the experimental tests are analysed to recognize different aspects of material behaviour. Then the appropriate constitutive model is developed and implemented into numerical procedures. The model parameters are identified on the basis of the available experimental data. Finally some benchmark simulations are performed.

Keywords: low cycle fatigue, thermoplastic coupling, damage, constitutive modelling

1. Introduction

Thermomechanical low cycle fatigue is one of the dominant failure modes in high temperature structural components, such as electric power boilers, boiler pipes, engine elements etc. Many studies show the complexity of the cyclic elasto-visco-plastic-damage behaviour, such as effect of non-proportional loading, cyclic softening/hardening behaviour dependent on strain amplitude and on the loading path, plastic strain range memorization effect, damage and so on [1].

Numerical analysis of low cycle fatigue concerns two main aspects: (1) developing algorithms for modelling coupled dissipative phenomena in nonisothermal conditions, and (2) identifying temperature dependent model characteristics on the basis of available experimental data. Both aspect will be regarded in the presented analysis.

2. Material behaviour

Preliminary experimental tests were performed on P91 steel specimens. The results (see Fig. 1) indicate that tested steel exhibits *cyclic softening*, regardless of the testing temperature [4]. This softening could be divided into three phases, which are: the *rapid softening* phase, followed by a *slow quasi-linear softening* phase, and finally again *fast softening till rupture*, being a consequence of micro-damage development in the material that ultimately causes failure of the tested sample, and is accompanied by the (characteristic for damage, cf [3]) hysteresis loop shape change from convex to concave (Fig. 1).

3. Constitutive Modelling

In the constitutive modelling of thermo-inelastic coupling, based on thermodynamics of irreversible processes with internal state variables, special attention was paid to the following problems:

1. Choice of state variables related to physical phenomena regarded in modelling, and formulation of proper state functions (state and dissipation potentials).
2. Derivation of model equations, regarding for thermo-mechanical coupling in evolution equations of state variables and thermodynamic forces conjugated to them, loading/unloading conditions and heat balance equation.
3. Proving thermodynamic admissibility (fulfilment of Clausius-Duhem inequality).

The constitutive model was derived which is able to describe properly the material behaviour in nonisothermal conditions and the experimentally observed features of low cycle fatigue process (see Figs 1 and 2, cf [2]).

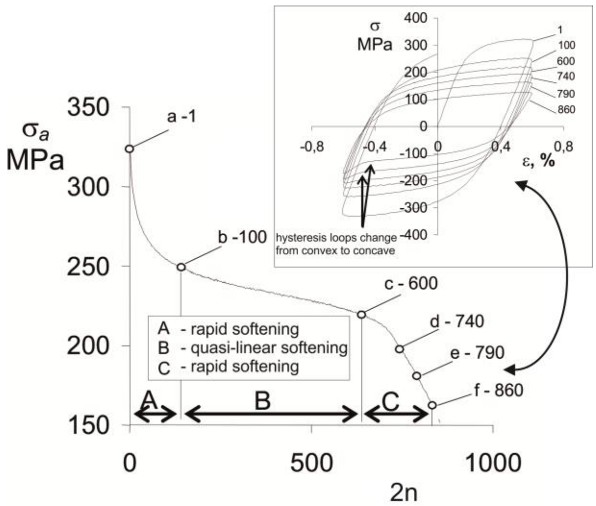

Figure 1: Maximal stress on cycle versus number of cycle (ε_{ac} =0,6% and test temperature $T = 600^0\mathrm{C}$).

4. Numerical implementation

4.1. Algorithms

The numerical algorithms applied in the presented analysis are based on the generalized mid-point rule. The fully implicit backward Euler scheme was chosen, which is always stable and very accurate. Adopting the Newton-Raphson method, the iterative solution procedure is defined, summarized in Table 1, where $\Delta\mathbf{S}$ is the vector containing the increments of the unknowns, $[\mathbf{J}] = \partial\mathbf{R} / \partial\Delta\mathbf{S}$ is the Jacobian matrix and $\mathbf{R}$ is a residual vector, containing the components $R_{S_i} = \Delta S_i - \Delta\hat{S}_i$, where ΔS_i is a variable while $\Delta\hat{S}_i$ denotes the function resulting from the evolution rule for *i*-th variable S_i. The iteration procedure is stopped when the norm of $\mathbf{R}$ is sufficiently small.

Table 1. Numerical algorithm.

Initialize:		$\Delta\mathbf{S}_n^{(0)} = \mathbf{0}$, $\mathbf{S}_{n+1}^{(0)} = \mathbf{S}_n$
Iterate:		DO UNTIL $\left\|\mathbf{R}(\mathbf{U}^{(k)})\right\| < TOL$ $k \leftarrow k+1$
	Compute iterate $\Delta\mathbf{S}^{(k+1)}$	$\Delta\mathbf{S}^{(k+1)} = \Delta\mathbf{S}^{(k)} - \left[\mathbf{J}^{(k)}\right]^{-1}\mathbf{R}(\Delta\mathbf{S}^{(k)})$
	Update $\mathbf{S}$	$\mathbf{S}_{n+1}^{(k+1)} = \mathbf{S}_n + \Delta\mathbf{S}^{(k+1)}$

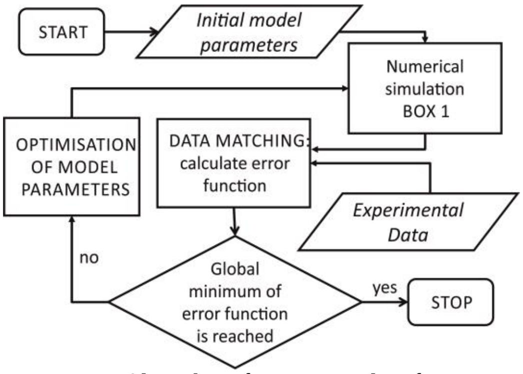

Figure 2: Algorithm of parameter identification.

4.2. Coupling effects

Our preliminary investigations show that the "sensitivity" of the modeling to the thermo-mechanical coupling effects depends both on the constitutive formulation and the numerical algorithm used for the implementation of mathematical equations into the numerical procedures [2]. For example, if the thermo-mechanical coupling terms are disregarded, the solution may be not much influenced, provided that the consistency condition is not enforced (only the yield condition is directly considered – *direct approach*) and the algorithmic tangent stiffness is not used. In this case the differences may result from the equation of heat balance (in which the derivatives of thermodynamic forces after temperature are involved) and from different ways of updating the results at the end of the step. *Indirect methods* make use of the consistency conditions, in which the kinetic equations of thermodynamic forces are necessary. In such cases the thermo-mechanical coupling terms play an important role both for proper evaluation of consistency multiplier and proper recognition of active/neutral/passive loading. Often the quantities at the end of the step are updated in an incremental way (*fully incremental approach*). In all such cases (both direct and indirect approaches) neglecting coupling terms may result in substantial errors. For the quadratic convergence of the Newton-Raphson procedure it is necessary to derive the *algorithmic tangent stiffness*, which depends on the inelastic load increment. The derivation of algorithmic tangent stiffness matrix also involves kinetic equations of thermodynamic forces, and disregarding coupling terms will result in errors.

4.3. Parameter identification

There are two ranges of temperature, in which the fatigue behavior of steel is different. Above the tempering temperature a sharp ageing is observed. In such case, the changes of mechanical properties of steel are induced by factors related independently to the history of temperature, therefore the identification of model parameters should be done all temperatures together. On the other hand, below tempering temperature the ageing remains nearly constant, so that the mechanical properties do not depend on the history of temperature. In such case it is sufficient to use several isothermal tests, and introduce the influence of temperature on the material parameters by interpolation techniques with polynomial or spline functions.

In the case of cyclic loading the identification process is time-consuming, therefore it is very important to reduce as much as possible the dimension of the field in which the optimal solution is searched. For example, the initial yield stress and the elastic modulus may be identified manually, considering monotonic tensile test (see Fig. 2).

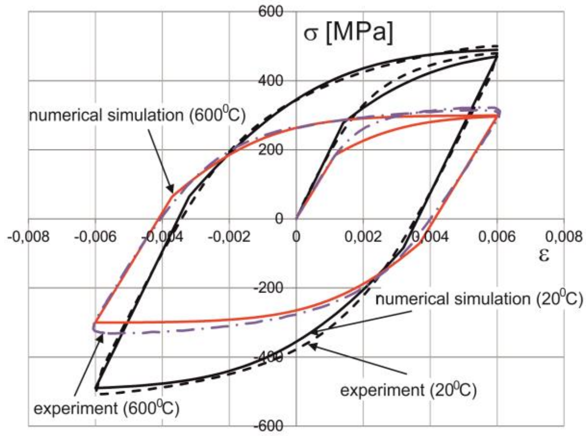

Figure 3: Comparison between experimental and calculated responses with a strain rate of 10^{-2} at temperatures of 20^0C and 600^0C.

5. Conclusions

In the present paper the algorithm of thermomechanical fatigue behaviour analysis is presented. Such analysis consists of five main steps: (1) experimental testing in several test temperatures, (2) constitutive modelling of material behaviour regarding the effects of temperature change, (3) numerical implementation of the mathematical model, (4) identification of model parameters in different test temperatures, to obtain temperature-dependent material characteristics, and (5) validation of the analysis by comparison between experimental and numerical results (see Fig. 3).

References

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