Vibration based methods for damage detection of plates

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Abstract

Vibration based methods are among the most popular approaches for health monitoring of structures. In this work several methods for damage detection based on the changes of the modal properties or in the forced response of plates are presented. The intact and damaged plates are discretized by the Finite Element Method. Damage indexes based on the change of the modal properties are constructed and its capability to predict the presence of damage and its location is presented. All these methods are compared with the damage detection method based on the analysis of the Poincaré maps of the motion of the structures. An improved version of this method developed already for beams is developed and tested numerically. Conclusions concerning the advantages and the applicability of the considered methods are deduced..

Keywords: vibration,damage detection, plates. ***Poincaré*** *maps, modal displacements, modal curvatures.*

1. Introduction

Damage generated within a structure usually leads to changes in the dynamic characteristics of the structure. This fact is a base of the vibrational based structural health monitoring methods. The easiest measurable characteristics of a structure are its eigen frequencies. As far as damage is in most cases a local phenomenon, its influence on the global dynamic characteristics of the structure, such as lower natural frequencies, might be weak for the most cases (Ref. [1,2]). The modes of vibrations are more sensitive to damages. The difficulty for their use comes from the fact that a lot of sensors have to be used. However, development of the experimental hardware and software gives new possibilities to measure the modal displacements with high accuracy. The contactless methods by using wireless sensor networks or scanning laser Doppler vibrometer enable measurement of displacement in a large number of nodes which increases practical applicability of the damage detection based on modal analysis. On the other hand the forced response depends on all geometrical and physical properties of the structure and the presence of a defect would influence the dynamic behaviour of the structure. If the structure is subjected to intensive dynamic loading leading to large amplitude vibrations the response will be even more sensitive to the presence of damage. The influence of the imperfections in the geometrical or physical properties is essential in the case of large amplitude vibrations and/or when oscillations are unstable or chaotic. In our previous work Ref. [3, 4] several modal based method were tested numerically and experimentally for damaged beams. The developed by us Poincare map based method based on the analysis of the phase space of the response of the beams Ref. [1,2] were also tested. In Ref. [4] an improved version of this method was suggested.. In the present work some known modal base methods – modal displacement method, modal curvatures method and the modal strain energy method will be compared in the case of damaged rectangular plates. They are compared with two, developed by us, forced response methods based on the Poincaré map of the response of the structure.

2. Basic equations

The geometrical nonlinear version of the Raisner-Mindlin plate theory is used to describe the plate motion. The governing equations are given below – Eqn (1). In this equations the geometrical nonlinear terms are grouped on the right hand side of the equation together with the external forces. In these equations where $u(x,y,t)$ and $v(x,y,t)$ are the in-plane displacements, w(x,y,t) is the transverse displacement and $\psi_x(x,y,t)$, $\psi_y(x,y,t)$ are the angles of the rotation of the normal of the cross section to the plate mid-plane. E is the Young modulus, G is the shear modulus, ν is the Poison ratio, k^2 is a shear correction factor which is assumed equal to 5/6 throughout the paper, h is the plate thickness a and b are the dimension of the plate in x and y directions correspondingly.

The boundary conditions are accepted clamped-clamped and in-plane fixed. This means that all displacements u, v and w and angular rotations ψ_x and ψ_y are zero along the boundaries.

If the considered case is without external loading (i.e. $p(x,y,t)=0$) the geometrical nonlinearities are not taken into account then the Eqns (1 c- e) will present the free vibrations of the plate.

As a results of the solution of the free vibration problem we obtain sets of normal modes $\mathbf{d}_n = \{\psi_{xn}, \psi_{yn}, w_n\}^T$ and $\mathbf{d}_n^* = \{\psi_{xn}^*, \psi_{xn}^*, w_n^*\}^T$, n=1,2, …,N for the undamaged (healthy) and damaged cases, correspondingly.

The plate was discretized by the Finite Element Method (FEM), by using linear quadrangular `shell elements. Two cases of damage were considered –reduced stiffness in a small part of the plate and reduced thickness in a small part.

Seven first natural frequencies and normal modes of vibrations were calculated for healthy and damaged plates. Then damage indexes based on the modal displacements, modal curvatures and strain energy were calculated. The presence and location of damage was shown by plotting these indexes as 3D plots or contours plots.

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$$
\begin{aligned}
&\frac{\partial}{\partial x}\left[A\left(\frac{\partial u}{\partial x}+\nu\frac{\partial v}{\partial y}\right)\right]+\frac{\partial}{\partial y}\left[\frac{(1-\nu)A}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]+ph\ddot{u}=G^{u}\ ,\ \frac{\partial}{\partial y}\left[A\left(\frac{\partial v}{\partial y}+\nu\frac{\partial u}{\partial x}\right)\right]+\frac{\partial}{\partial x}\left[\frac{A(1-\nu)}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right]+rh\ddot{v}=G^{v}\\
&\frac{\partial}{\partial x}\left(D\left[\frac{\partial\psi_x}{\partial x}+\nu\frac{\partial\psi_y}{\partial y}\right]\right)+\frac{(1-\nu)}{2}\frac{\partial}{\partial y}\left(D\left[\frac{\partial\psi_x}{\partial y}+\nu\frac{\partial\psi_y}{\partial x}\right]\right)-\frac{(1-\nu^2)k^2A}{2}\left(\psi_x+\frac{\partial w}{\partial x}\right)+c_2\dot{\psi}_x+\frac{\rho h^3}{12}\ddot{\psi}_x=0\\
&\frac{\partial}{\partial y}\left(D\left[\frac{\partial\psi_y}{\partial y}+\nu\frac{\partial\psi_x}{\partial x}\right]\right)+\frac{(1-\nu)}{2}\frac{\partial}{\partial x}\left(D\left[\frac{\partial\psi_y}{\partial x}+\nu\frac{\partial\psi_x}{\partial y}\right]\right)-\frac{(1-\nu^2)k^2A}{2}\left(\psi_y+\frac{\partial w}{\partial y}\right)+c_2\dot{\psi}_y+\frac{\rho h^3}{12}\ddot{\psi}_y=0\\
&\frac{(1-\nu)k}{2}\left\{\frac{\partial}{\partial x}\left(A\left[\psi_x+\frac{\partial w}{\partial x}\right]\right)+\frac{\partial}{\partial y}\left(A\left[\psi_y+\frac{\partial w}{\partial y}\right]\right)\right\}+c_1\frac{\partial w}{\partial t}+\rho h\ddot{w}=-p+G^{L}
\end{aligned}
\qquad \text{(1 a-e)}
$$

where

$$
G^{u}=-0.5\frac{\partial}{\partial x}\left\{A\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]\right\}-0.5\frac{\partial}{\partial y}\left\{A(1-\nu)\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right\}),\ G^{v}=-0.5\frac{\partial}{\partial y}\left\{A\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]\right\}-0.5\frac{\partial}{\partial x}\left\{A(1-\nu)\left(\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right\}
$$

$$
G^{L}(x,y,t)=-\left(N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}\right)
$$

Then, the plate was subjected to a harmonic loading. The cases when the excitation frequency was equal to the first natural frequency and the case when the frequency is equal to the half of the first natural frequency were considered. The pseudo-normal mode superposition Ref [2] was used to obtain the nonlinear forced response of the plate. The Poincare maps of the forced responses of the damaged and healthy plates were calculated and the Poincare map based damage index and the Improved Poincare map based damage index were constructed.

3. Numerical results and conclusions

The numerical results demonstrate the applicability of the considered methods for damage detection. From the modal based method the best results show the method based on the modal strain energy. The 2D modal curvatures based methods also predict very well the presence and the location of damage. The based damage method and the improved Poincaré map based method also demonstrate good capability to predict damage location (see Fig.1).

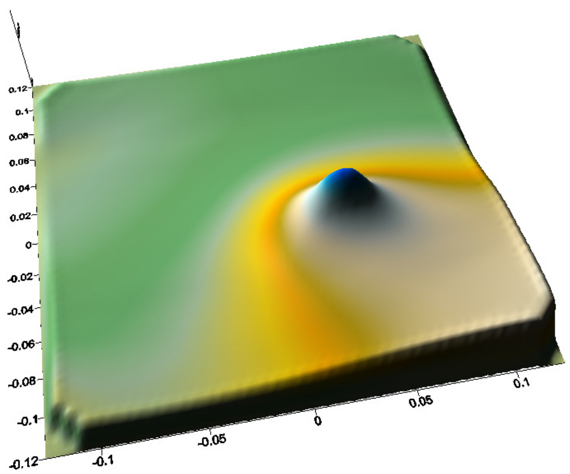

Figure 1: Poincaré map based Damage index for plate with reduced thickness in a small part of its surface.

The 3D plots of the new improved Poincaré map based method show very sharp pick at the place of the damage. This method is tested for the first time for plate structures and it gives excellent results in fault detection procedures, in both cases of reduced stiffness and reduced thickness. These results show that it could be applied for on-line structural health monitoring.

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