Bifurcation and stability analysis of a nonlinear milling process

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Abstract

Numerical investigations of milling operations dynamics are presented in this paper. A two degree of freedom nonlinear model is used to study workpiece-tool vibrations. The analysed model takes into account both, flexibility of the tool and the workpiece. The dynamics of the milling process is described by the discontinuous ordinary differential equation with time delay, which can cause process instability. First, stability lobes diagrams are created on the basis of the parameters determined in impact test of an end mill and workpiece. Next, the bifurcations diagrams are performed for different values of rotational speeds.

Keywords: chatter, bifurcations, nonlinear vibrations, time delay, stability

1. Introduction

Milling is one of the most popular methods of manufacturing, therefore an increase of its productivity as well as final product quality is still an important task in engineering practice. During machining, at specific combination of cutting depth and spindle speed, the regenerative effect may arise. Machine tool vibrations have a negative effect on the quality of a machined surface of a workpiece.

In milling the cutting tool rotates producing time-periodic coefficients as well as time-delay in the mathematical model. Therefore, the model of milling is described by means of time-periodic delay-differential equations (DDEs) [1]. The delayed state feedback is commonly used to model machining processes with, so called, a regenerative effect that is very harmful because it can produce self-excited chatter vibrations [3,6]. Nonlinear models, which generate the chatter phenomenon, have received much attention in recent years [2]. Stability theories based on linear dynamics cannot predict some of the more interesting phenomena in the milling process. In nonlinear dynamics several techniques such as bifurcation and perturbation analysis are used. Also, an analysis of nonlinear regenerative chatter using a power series for cutting force was carried out [4].

In this analysis a two degree of freedom nonlinear model of milling is presented. The parameters applied in the numerical simulations are received on the basis of modal analysis of the real experimental system. The stability lobes and bifurcations diagrams of the milling process for different values of parameters are determined showing various bifurcation scenarios depending on nonlinear parameter.

2. Model of milling

Models of a milling process are non-smooth by nature because a cutting tool has several cutting teeth, which are in contact with a workpiece during some time intervals of cutting. For the rest of time, the cutting edge is not in contact with the workpiece. This causes discontinuities, which make difficulties in numerical simulations and preclude analytical solutions. Therefore, modelling process of milling is rather difficult and complicated from analytical point of view. Here, a step function is used to model discontinuities. The model takes into account the susceptibility of the tool and the workpiece in the feed direction. Therefore, the 2dof model, described by following equations, is used

$$\begin{aligned} &\ddot{x}_1(t)+2\zeta_1\omega_{n1}\dot{x}_1(t)+\omega_{n1}^2x_1(t)+\gamma_1x_1(t)^3=\frac{1}{m_1}\sum_{p=1}^{z}F_p(t)\\ &\ddot{x}_2(t)+2\zeta_2\omega_{n2}\dot{x}_2(t)+\omega_{n2}^2x_2(t)+\gamma_2x_2(t)^3=-\frac{1}{m_2}\sum_{p=1}^{z}F_p(t)\end{aligned} \tag{1}$$

where, ζ_1 and ζ_2 are damping coefficient, γ_1 and γ_2 are nonlinear stiffness coefficient, ω_{n1} and ω_{n2} are the linear natural frequencies of the tool and workpiece. The resultant cutting force caused by the *p-th* tooth ($p=1,2...,z$) in the x direction is given by the approximate equation

$$F_p(t)=g_p(t)\left[-F_{tp}(t)\cos\theta_p(t)-F_{np}(t)\sin\theta_p(t)\right] \tag{2}$$

The cutting force F_p depends on the angular tool position θ_p of the *p-th* cutting tooth and consists of the tangential F_{tp} and the normal F_{np} force component. z means a number of tool teeths, g_p defines when *p-th* tooth is active (cuts a material). The tangential and radial cutting force acting on the tool are proportional to the axial depth of cut b and chip width w_p according to the equations

$$F_{tp}(t)=K_tbw_p(t)^\kappa,\ F_{np}(t)=K_nbw_p(t)^\kappa \tag{3}$$

K_t and K_n are the specific cutting forces which depend on the cutting material properties. Typical relationship between K_t and K_n for classical materials is $K_n=0.36K_t$. The coefficient κ also depends on the nonlinear material properties [5], and is usually estimated from *0.75* to *1*. In this paper an influence of κ is focused on. The chip width $w_p(t)$ is a function of the feed f, the present tool and workpiece vibrations $x(t)$ and the previous tool and workpiece position $x(t-\tau)$ as follows.

$$w_p(t)=\left[f+\left(x_1(t)-x_2(t)\right)-\left(x_1(t-\tau)-x_2(t-\tau)\right)\right]\sin\theta_p(t) \tag{4}$$

where, $\tau=60/zn$ is the tooth passing period, n means rotational speed of the tool (in rpm).

3. Stability of milling process

Basing on differential equations of motion (1) the numerical simulations were performed in the Matlab - Simulink package

* Financial support of Development of Young Researchers Program 30/MN/2016 is gratefully acknowledged.

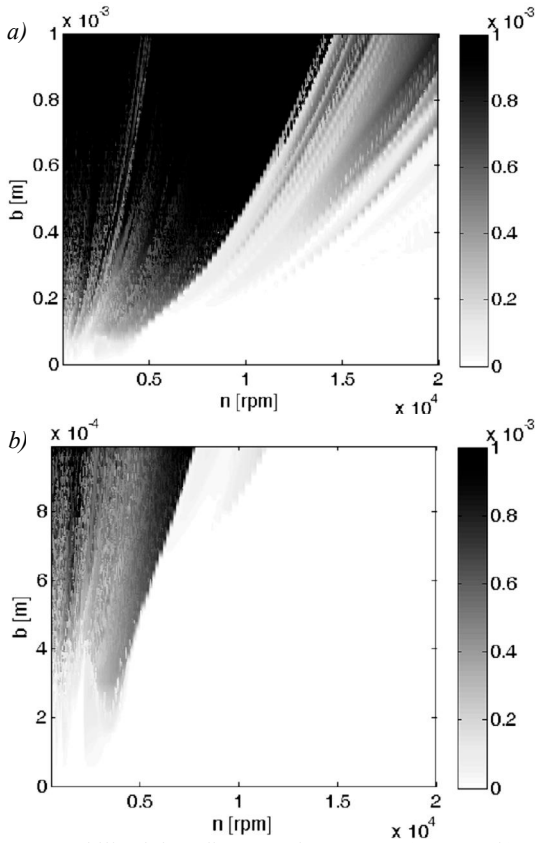

Figure 1: Stability lobes diagrams for *κ=0.75* a), *κ=0.9* b).

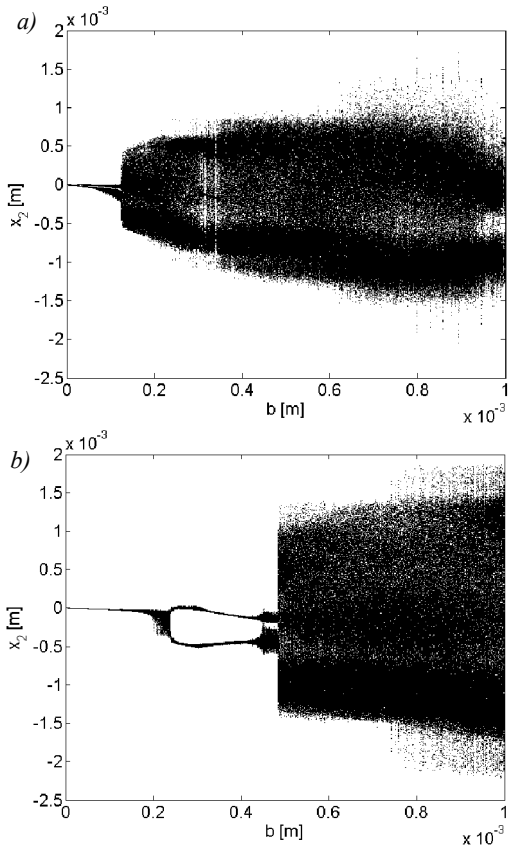

Figure 2: Bifurcation diagrams for *κ=0.75* and *n=5000rpm* a), *n=10000rpm* b).

Table 1: Parameters of milling model

Value of parameters			
ω_{n1}	865.43 [rad/s]	K_n	160 [MPa]
ζ_1	0.0406 [-]	K_t	450 [MPa]
ω_{n2}	318.93 [rad/s]	f	0.01 [mm/tooth]
ζ_2	0.0396 [-]	z	4 [-]
γ_1, γ_2	2e12 [N/m^3]		

using the Runge-Kutta method of fourth order with variable step of integration. Parameters of the model were obtained from a modal analysis carried out on a real test system (Tab. 1). The stability lobes diagrams of milling process are plotted for two values of *κ* coefficient (Fig.1). Stability diagrams are presented in a grayscale dependent on amplitude of vibrations which are defined by the displacement between the tool and the workpiece (x_1-x_2). Light areas illustrate a stable milling process, where the chip width is a constant. In a further part of the research bifurcation analysis is performed, where depth of cut *b* is used as a bifurcation parameter (Fig.2). Two values of tool rotational speed are applied, *n=5000 rpm* and *n=10000 rpm*. Figure 2 presents an effect of changes in the depth of cut *b* on dynamics of the milling process. For tool rotational speed *n=5000 rpm* (Fig. 2a) one can observe a gradual increase of vibrations in the unstable area, including chaotic vibrations. For tool rotational speed *n=10000 rpm* (Fig. 2b) additionally periodic oscillations in the range of *b=0.25-0.4mm* are observed.

4. Conclusions

The results of numerical studies of nonlinear milling model with two degrees of freedom are presented in the paper. By means of parameters obtained experimentally, stability lobes diagram for two values of *κ* coefficient are determined. It has been observed that with the increase of *κ* the stable areas expand to become larger. Based on bifurcations analysis it can be concluded that the system shows a different dynamic behavior for various cutting speed and depth of cut. Moreover, an influence of *κ* on bifurcation scenario is a new finding.

References

[1] Altintas, Y. and Lee, P.A., General mechanics and dynamics model for helical end mills, *CIRP Annals-Manufacturing Technology,* 45 (1), pp. 59-64, 1996.

[2] Balachandran, B., Nonlinear dynamics of milling processes, *Philosophical transactions of the royal society A*, 359, pp. 793-819, 2001.

[3] Bobrenkov, O.A., Khasawneh, F.A., Butcher, E.A. and Mann, B.P., Analysis of milling dynamics for simultaneously engaged cutting teeth, *Journal of Sound and Vibration*, 329, pp. 585-606, 2010.

[4] Stepan, G., Insperger, T. and Szalai, R., Delay, parametric excitation and the nonlinear dynamics of cutting process, *Journal of Bifurcation and Chaos*, 15 (9), pp. 2783-2798, 2005.

[5] Insperger, T., Stepan, G., Bayly, P.V. and Mann, B.P., Multiple chatter frequencies in milling processes, *Journal of Sound and Vibration*, 262, pp. 333-345, 2003.

[6] Weremczuk, A., Kecik, K., Rusinek, R. and Warminski, J., The dynamics of the cutting process with duffing nonlinearity, *Maintenance and Reliability,* 15 (3), pp. 209-213, 2013.