Design of additively manufacturable least compliant structures

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Abstract

The paper presents the process of 3D printing of planar structural members satisfying the condition of the minimal compliance among all the members transmitting a given force system to a given support, within a prescribed design domain. The design process consists of several steps including: solving the minimum compliance problem within the Young Modulus Design method, the inverse homogenization and forming the programs which steer the printer.

Keywords: Young Modulus, optimal design, homogenization, additive manufacturing

1. Introduction

The optimum design problem of construction of the least compliant structures leads to composite like solutions: the optimal designs are endowed with a spatially varying underlying microstructure. The recent rapid progress in 3D printing paves the way towards making such theoretical designs manufacturable, see e.g. Smith et al. [1]. The present paper shows a procedure from the very formulation of the 2D minimum compliance problem towards the final design produced by the additive manufacturable process. We make use of the YMD method of optimal distribution of the Young modulus, while keeping the Poisson ratio constant, see Ref. [3]. The design process consists of the three main steps:

1. Introducing one parameter constructive model of inverse homogenisation
2. Finding optimal distribution of elastic moduli minimizing the compliance
3. Dividing the design domain into cells and filling them with substructures generated by model from item 1.

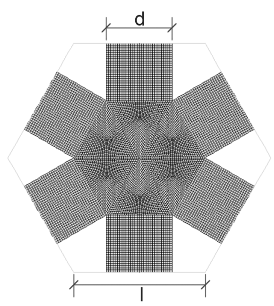
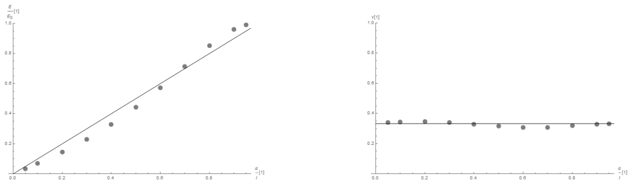


Figure 1: Example mesh: 2760 instances of 8-node element

2. Inverse homogenization: recovery of isotropic microstructures

The inverse homogenization method applied here is aimed at reconstructing isotropic microstructures characterized by given values of Young's moduli. We make use of specific features of a hexagonal basic cell, see Fig.1, whose effective Young's modulus is parameterized by the ratio $\frac{d}{l}$, being the relative width of the ligament. The effective elastic moduli tensor corresponding to the hexagonal cell is isotropic, with its effective Poisson's ratio being practically independent of the $\frac{d}{l}$ ratio in the whole range: $[0, 1]$, see Łukasiak [2]. One can prove that for $\frac{d}{l}$ tending to zero the effective Poisson ratio tends to $\frac{1}{3}$. We make use of this specific feature of the microstructure and assume that Poisson's ratio of the source material is also equal to $\frac{1}{3}$. Consequently, the effective Poisson ratio is practically equal to $\frac{1}{3}$ for the whole range of $\frac{1}{3}$ ratio. Fortunately, a wide class of 3d printable materials satisfies this requirement.



(a) Young modulus (b) Poisson ratio

Figure 2: Results of solving a set of homogenisation tasks for selected values of $\frac{d}{l}$

Let us stress that the inverse homogenization used here, based on the continuum modelling, circumvents possible modelling difficulties of the alternative lattice-based modelling, like the problem of the material overlapping at nodes.

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3. Young Modulus Design (YMD)

The choice of topological optimization method has to correspond to capabilities of available additive manufacturing process. We decided to use Young Modulus Design (YMD) Ref. [3] method so as to keep the space of microstructures one dimensional, parametrized only with the Young moduli. YMD is a version of Free Material Design, where the space of available Hooke tensors is reduced to isotropic materials with fixed Poisson ratio.

The available free material design (FMD) methods lead to the auxiliary problem

$$Z = \min_{\boldsymbol{\tau}\in\Sigma_T(\Omega)} \int_\Omega |||\boldsymbol{\tau}|||\mathrm{d}\boldsymbol{x}, \tag{1}$$

where $\Sigma_T(\Omega)$ stands for a set of statically admissible stresses and $|||\cdot|||$ is a norm specified by the space of allowed Hooke tensors. In the YMD the norm involved in the integrand of (1) reads

$$|||\boldsymbol{\tau}||| = \sqrt{\frac{3-\nu}{2(1+\nu)}(\mathrm{tr}\boldsymbol{\tau})^2 + \frac{3-\nu}{1-\nu}\|\mathrm{dev}\boldsymbol{\tau}\|^2}. \tag{2}$$

The problem discussed here is to find an optimal bracket capable of transmitting the surface load of the horizontal resultant, applied along the half of the boundary of a circle, towards two circular supports, see Fig. 3. The design domain outstretches at least $0.35a$ in every direction from the circles creating a rectangle $1.35a \times 2a$.

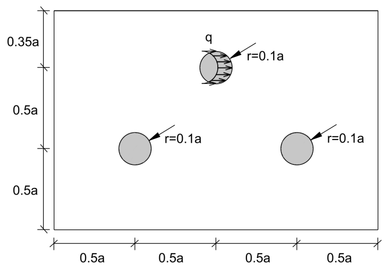

Figure 3: Posing a representative problem

Due to the integrand in (1) having a linear growth the solution to the auxiliary problem (1) may vanish on a certain part of the design domain. The effective domain of the minimizer is just the domain to be occupied by the optimal isotropic and nonhomogeneous material. It is of vital importance to predict the initial design domain sufficiently large, to make the design independent of this choice.

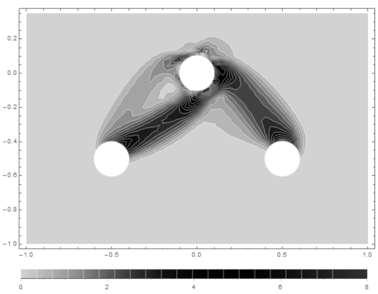

Figure 4: Optimal distribution of Young modulus

4. Building discretized model

The continuum distribution of Young's modulus obtained from the YMD method needs postprocessing to show the microstructure to be printed. This postprocessing is performed with using the inverse homogenization technique applied to the structure of hexagonal representative volume elements, as discussed in Sec. 2. The design domain is divided into hexagonal cells. At each cell the relative density of the material is defined, corresponding to the Young modulus value at the center of the cell. If the computed thickness of the ligament is less than the admissible value by the 3D printer's resolution, the cell of such thin ligaments is removed.

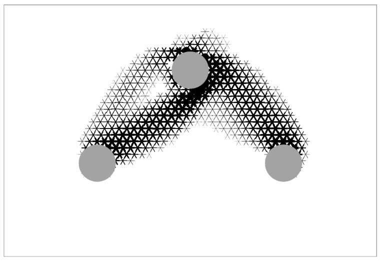

Figure 5: Model based on results from Figure. 4

5. 3d printed element and final remarks

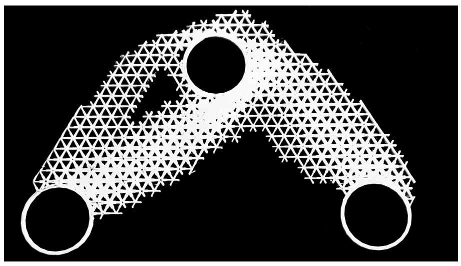

Figure 6: Photography of a printed element

The proposed method allows to develop automatic system supporting designers to create models of lightweight structures, providing significant stiffness under a single load case.

Current paper shows that YMD, being a structural topology optimization method brings information useful for designers, although the method provides continual distribution of elastic properties of the optimal material and does not define the construction.

References

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