Determination of post-shakedown quantities of a pipe bend via the Simplified Theory of Plastic Zones compared with load history dependent incremental analysis

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Abstract

The Simplified Theory of Plastic Zones (STPZ) may be used to determine post-shakedown quantities such as strain ranges and accumulated strains. The principles of the method are summarized succinctly and the practical applicability is shown by the example of a pipe bend subjected to internal pressure and cyclic in-plane bending.

Keywords: Simplified Theory of Plastic Zones, pipe bend, cyclic loading, multiaxial ratcheting, post-shakedown quantities

1. Introduction

In case of over-elastic cyclic loading, strain ranges and accumulated strains are needed for performing a lifetime analysis. If an incremental analysis is used for obtaining these post-shakedown quantities, many load cycles must be analysed on a step-by-step basis, if a ratcheting mechanism is present. The computational effort involved can easily exceed time and hardware resources. Based on Zarka's method, the Simplified Theory of Plastic Zones was developed as a direct method, aiming at obtaining post-shakedown quantities regardless of the load history. The computational cost consists of a handful of pure elastic FE calculations. The reward is a well-approximated solution.

The Simplified Theory of Plastic Zones (STPZ) is described in detail in [1], including many examples.

2. Basics about the STPZ

In simple words, the STPZ is used to search residual stress fields in the state of elastic or plastic shakedown, based on pure elastic calculations. Once the constant or cyclically varying residual stress fields are known, elastic-plastic stresses, strains, displacements etc. can subsequently be determined by simple superposition with fictitiously elastic calculated stresses, strains (or strain ranges), displacements etc.

To obtain the residual stress field, a "modified elastic analysis" has to be performed, where the elastic material parameters (Young's modulus E and Poisson's ratio ν) of the structure are modified in the plastic zone, along with applying some appropriately defined initial strains as modified loading. The modified material parameters are independent of the load level, while the initial strains are not.

Mises yield surface and multilinear kinematic hardening are adopted. Multilinear kinematic hardening is introduced by employing an overlay model [2]. If reduced to a bilinear material model, unlimited linear kinematic hardening is obtained, so that the modified elastic material parameters become

$$E^{\text{mod}} = E_t\,,\ \nu^{\text{mod}} = \frac{1}{2} - \frac{E_t}{E}\left(\frac{1}{2} - \nu\right), \tag{1}$$

whereby E_t means the elastic-plastic tangent modulus. The initial strain components $\varepsilon_{i,0}$ are given by

$$\varepsilon_{i,0} = \frac{3}{2}\frac{Y_i}{C}\,,\ C = \frac{EE_t}{E - E_t}\,, \tag{2}$$

where use of the so-called transformed internal variable Y_i is made which is defined by the difference of the deviatoric part of the residual stress ρ_i and the backstress ξ_i:

$$Y_i = \xi_i - \rho_i^{'}\,. \tag{3}$$

None of these stresses are known a priori. Providing reasonable estimates of Y_i on the basis of fictitious elastic analyses is the main challenge for the STPZ. Once the geometry of the plastic zone as well as the values of Y_i in the plastic zone are estimated, a modified elastic analysis is performed, of which the results can be used to improve the initial estimate of the geometry of the plastic zone and of Y_i. Thus, the entire procedure is subject to iterative improvement.

Usually, convergence is reached within few modified elastic analyses, and the quality of the results of the converged solution approximates the solution obtained by cyclic incremental analyses very well. Strain ranges in the state of plastic shakedown and strains accumulated through many cycles of loading prior to elastic or plastic shakedown can thus be achieved at little computational effort, just consisting of some linear elastic analyses (fictitious elastic and modified elastic analyses).

3. Example of a pipe bend

In thermal power plants, pipe bends are loaded by an in-plane bending moment $M_{\text{in-plane}}$ resulting from thermal expansion of adjacent straight pipe sections. The emerging cyclic loading, along with the primary load (pressure p), can produce a ratcheting mechanism.

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3.1. Simplified geometry model

Assuming that the stiffening effect of the attached straight pipes on the ovalisation of the pipe bend can be ignored (at least in the maximum stressed section of a 90° bend), the elbow can be considered as an axisymmetric torus shell, as shown in Fig. 2.

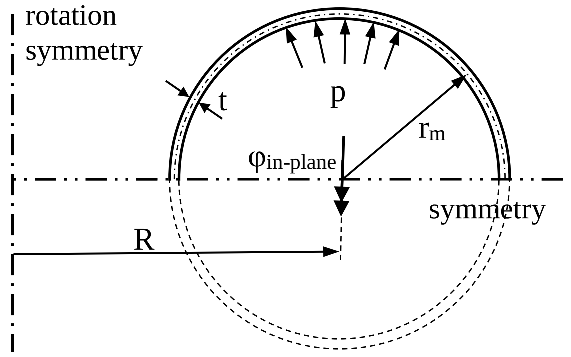

Figure 2: Pipe bend simplified as a torus section

3.2. Material model and loadings

The material model used is a bilinear model with linear kinematic hardening, as shown in Fig. 3. Internal pressure p is applied as steady load (80% of elastic limit load), while the displacement-controlled in-plane bending (three times the elastic limit load, corresponding to an angle of 1.325° related to a 90° elbow) is applied cyclically.

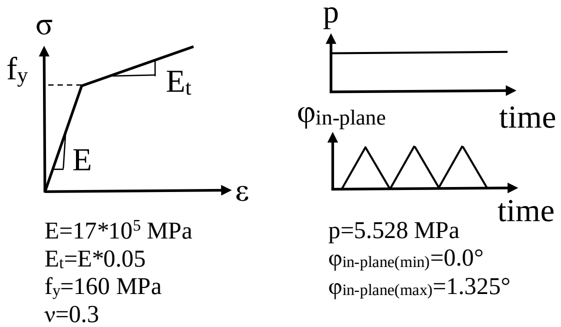

Figure 3: **Left:** material model, **right:** loading history

3.3. Results and conclusion

The advantage of the STPZ is a good approximation of accumulated strain with minimal computational effort, Fig. 4. Discrepancies between STPZ and incremental analysis are hardly noticeable. On a percentage basis the difference in circumferential strains amounts to 2.3%.

The left curve in Fig. 5 demonstrates a ratcheting mechanism, in which the accumulation of strains per cycle is vividly shown. In comparison, the left curves show the solutions of the STPZ for minimal and maximal loading, requiring significantly smaller computational effort.

In this example, about thousand equilibrium iterations were required by the incremental analysis to get through the 15 load cycles before plastic shakedown is approximately achieved, Fig. 5 left. In contrast, 22 linear elastic analyses (2 fictitious elastic analyses, 10 modified elastic analyses for determining the strain range and another 10 modified elastic analyses for determining the accumulated strain) were sufficient for the STPZ. Finally, it needs to be mentioned that each solutions of the modified analyses represents an approximation of the shakedown state, Fig. 5 right.

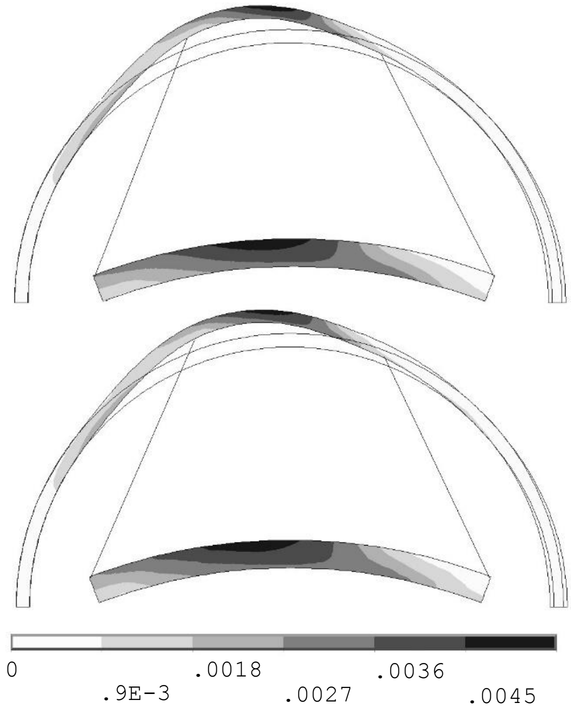

Figure 4: Contour plots of circumferential strains in the state of shakedown at zero in-plane bending, **top:** incremental analysis, **bottom:** Simplified Theory of Plastic Zones

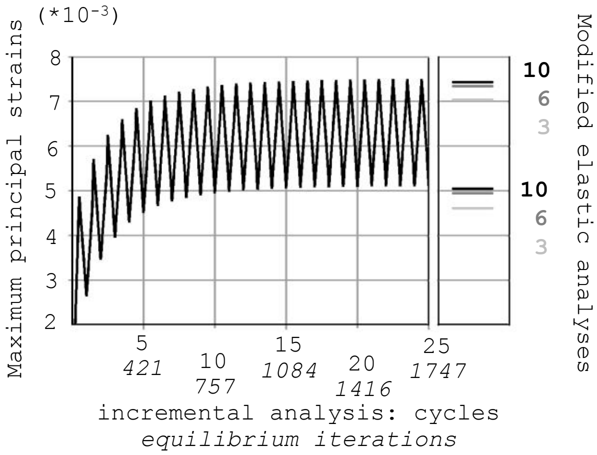

Figure 5: Maximum principal strains over computational effort, **left:** incremental analysis, **right:** STPZ

References

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[2] Hübel, H., Simplified Theory of Plastic Zones for cyclic loading and multilinear hardening, *Int. J. Press. Vessels Pip.*, 129-130, pp. 19-31, 2015.