Error driven remeshing strategy in an elastic-plastic shakedown problem

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Abstract

A shakedown based approach has been for many years successfully used to calculate the distributions of residual stresses in bodies made of elastic-plastic materials and subjected to cyclic loads exceeding their bearing capacity. The calculations performed indicated the existence of areas characterized by extremely high gradients in the stress field sought. In order to account for that, relatively dense nodal meshes had to be used during calculations in disproportionately large parts of considered bodies, resulting in unnecessary expenditure of computer resources. Therefore the effort was undertaken to limit the areas of high mesh densities and drive the mesh regeneration algorithm by selected error indicators.

Keywords: shakedown problem, error control, remeshing strategy, Finite Difference Approximation

1. Introduction

The residual stresses may arise in metallic materials as a result of the manufacturing process or the conditions encountered during service [6], when the applied loads exceed the elastic load envelope. In the case of residual stresses induced during service by localized cyclic contact loads of high intensity [7], the obtained residual stress distributions are characterized by very high gradients within and adjacent to the plastic zone. Thus in these zones relatively dense mesh is necessary in order to replicate the stress behaviour correctly. As these zones are not known a'priori, the usual procedure was to generate the sufficiently dense mesh in the whole potentially affected zone, thus ending in unnecessary long calculation times. The application of intelligent meshing algorithm should decrease this time significantly.

2. Mechanical model

The mechanical model allowing for the calculation of the stable (time independent) residual stress distribution in a prismatic body made of elastic-perfectly plastic material, subjected to the cyclic loading program exceeding its elastic bearing capacity had been proposed for the first time in [4]. Later on this model has been developed to account for kinematic hardening of the material [2] and used as a basis for development of three independent computational models based on the Finite Element [3], Boundary Element [1] and Generalized Finite Difference [5] methods. The final version of this model may be stated as the following two step procedure:

I calculate the correlation matrix A_{ijkl} :

$$\sigma_{ij}^{r} = A_{ijkl} \cdot \varepsilon_{kl}^{p} \tag{1}$$

solving the following nonlinear constrained optimization problem for self equilibrated stresses σ_{ij}^{r} as a function of plastic distortions ε_{ij}^{p} :

$$\min_{\sigma_{ij}^{r}} \Theta(\sigma_{ij}^{r}),\ \Theta(\sigma_{ij}^{r}) = \int_V \sigma_{ij}^{r} \cdot C_{ijkl} \cdot \sigma_{kl}^{r} \cdot dV - \int_V \varepsilon_{ij}^{p} \cdot \sigma_{ij}^{r} \cdot dV \tag{2}$$

at:

$$\sigma_{ij,j}^{r} = 0, \quad \text{in } V \text{ – internal equilibrium conditions,} \tag{3}$$

$$\sigma_{ij}^{r} \cdot n_j = 0, \quad \text{on } \partial V \text{ – static boundary conditions,} \tag{4}$$

II find the ε_{ij}^{p} which minimize the total complementary energy functional:

$$\min_{\varepsilon_{ij}^{p}} \Psi(\varepsilon_{ij}^{p}),\ \Psi(\varepsilon_{ij}^{p}) = \int_V \varepsilon_{gh}^{p} \cdot A_{ghij}^{T} \cdot C_{ijkl} \cdot A_{klmn} \cdot \varepsilon_{mn}^{p} \cdot dV \tag{5}$$

at:

$$\Phi\left(A_{ghij} \cdot \varepsilon_{ij}^{p} + c \cdot \varepsilon_{ij}^{pC} + \sigma_{ij}^{E} + \sigma_{ij}^{T}\right) - 1 \le 0 \tag{6}$$

in V – the yield conditions,

where:

$c = \dfrac{E \cdot H}{E - H}$, – kinematic hardening parameter.

The following denotations hold in (1) - (6):

- σ_{ij}^{r} – residual stresses arising in the considered body due to the actual applied cyclic loads exceeding the body's elastic bearing capacity,
- ε_{ij}^{p} – the sought plastic strains,
- ε_{ij}^{pC} – plastic strains in the computational plastic zone assumed to be constant at each point during the iteration, but changing between iterations,
- σ_{ij}^{E} – elastic stresses calculated as if the considered body deformed purely elastically under current loading program,
- σ_{ij}^{T} – thermal stresses induced in the considered body due to the external actions and determined as if the body deformed purely elastically under the considered thermal load,
- A_{ghij} – singular correlation matrix linking plastic strains and residual stresses (1),
- E – Young's modulus,
- H – elastic-plastic tangent modulus (hardening ratio),
- C_{ijkl} – elastic compliance matrix.

In order to determine the final plastic strain ε_{ij}^{p} and residual stress σ_{ij}^{r} state an iterative approach has to be used. At the beginning the initial plastic strain state $\varepsilon_{ij}^{pC} = 0$ is assumed. During the subsequent n iterations this state is modified according to the following formula:

$$ {}^{n}\varepsilon_{ij}^{pC} = \lambda \cdot {}^{n-1}\varepsilon_{ij}^{pC} + (1-\lambda) \cdot {}^{n-2}\varepsilon_{ij}^{pC}, \lambda \in [0,1] \tag{7} $$

until a stable plastic strain distribution ${}^{n}\varepsilon_{ij}^{p}$ is obtained.

This model has been used successfully to determine the residual stresses induced in prismatic bodies by contact loads. The Finite Difference Method generalized for arbitrarily irregular grids has been used to develop the appropriate numerical model with quadrilateral integration areas spanned between nodes (Fig. 1) and equilibrium as well as yield conditions enforced at the centroids of those areas.

3. Standard approach

As the contact loads generate highly localized residual stress distributions characterized by very high gradients rapidly changing sign over small zones, relatively dense computational meshes had to be used in the whole body (Fig. 1) resulting in unnecessary expenditure of time and computer resources.

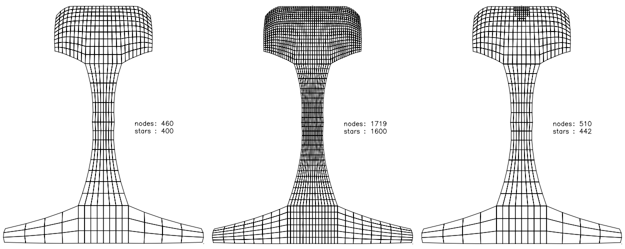

Figure 1: Meshes used for calculations: low density at left, high density in the middle and adapted at right.

4. Modified approach

An effort was undertaken to adapt the mesh to the solved problem by locally adjusting its density according to selected criteria. Two criteria have been considered so far, i.e. the straightforward one, where the integration areas in which plastic behaviour of material occured (i.e. the computational plastic zone) are split into four sub areas of similar size and then the calculations are rerun on thus modified mesh, and the more complex one, where a preset percentage of integration areas (those in which the complementary energy is the highest) is treated as in the first criterion. The first criterion should ensure a high quality of the solution (residual stresses) at least in the vicinity of the contact zone, where yielding occurs. The second criterion should work in a manner similar to the first one, but should offer the advantage of better control over the area occupied by integration zones designated for subdivision.

5. Numerical results

An 132RE (US) railroad rail subjected to simulated service load of 150 kN applied over a 1,25 by 2,00 cm rectangular patch at the axis of symmetry, as a biparabolic pressure distribution, has been treated as a test example.

The obtained σ_{xx}^{r} (horizontal in plane) residual stress distribution along the vertical axis of symmetry of the rail is depicted in Fig. 2 against three reference solutions obtained on relatively regular meshes, having 400 (Fig. 1, left), 1600 (Fig. 1, center) and 6400 integration elements, respectively. Vertical axis shows the distance from the rail foot (in cm), while the value of residual stress in MPa is shown on horizontal axis. The locally adjusted mesh had the density of the third mesh in the plastic zone and the first mesh outside this zone (Fig. 1, right).

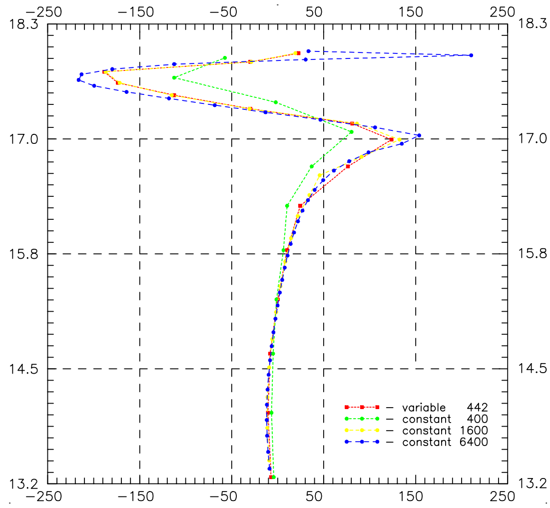

Figure 2: σ_{xx}^{r} residual stress along rail axis of symmetry.

6. Conclusions

Preliminary analysis seems to indicate that the residual stress distributions obtained on the mesh with locally modified density (Fig. 1 right) follow quite accurately the distributions obtained on the denser of meshes (Fig. 1 center) used as a basis for comparison.

The most up to date results will be shown during the conference.

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